

From Frequency Scan to Immittance-Based Stability Theory:

# Frequency-Domain Methods for IBR and Future Power Systems

Jian Sun

Rensselaer Polytechnic Institute

[jsun@rpi.edu](mailto:jsun@rpi.edu)

# Outline

- Theory
- Applications

# Impedance + Admittance → Immittance

- A Term First Used by Hendrik W. Bode



## Network Analysis and Feedback Amplifier Design

By  
HENDRIK W. BODE, Ph.D.,  
*Research Mathematician,*  
BELL TELEPHONE LABORATORIES, INC.

TENTH PRINTING



In view of the obvious analogy between the mesh and nodal methods of analyzing a circuit, the two methods will be used indifferently in most of the following discussion. The primes, which were used in equations (1-23) and (1-24) to distinguish the nodal determinants from those obtained from the mesh equations, will ordinarily be omitted. The determinant  $\Delta$  will thus be used to refer to either system unless there is some particular reason for distinguishing between them. The symbol  $W$ , which may perhaps be called an “adpedance” or “immittance,” will be used to refer to an element in either system.

D. VAN NOSTRAND COMPANY, INC.

TORONTO

NEW YORK

LONDON

# Frequency Scan

- A Method to Screen SSR Involving Generators
  - Synchronous Generators since 1970's; Type-III Turbines since ca. 2010
- Split System into Two Sides at Point of Interconnection
- Scan the Impedance of Each Side
  - Synchronous Generator Impedance can also be Modeled
  - No Models Available for Converters; Frequency Scan was the Only Option
- Evaluate SSR Potential Based on the Sum of Two Impedances
  - Negative Resistance at Reactance Crossover Frequency → SSR

# Frequency-Domain Methods

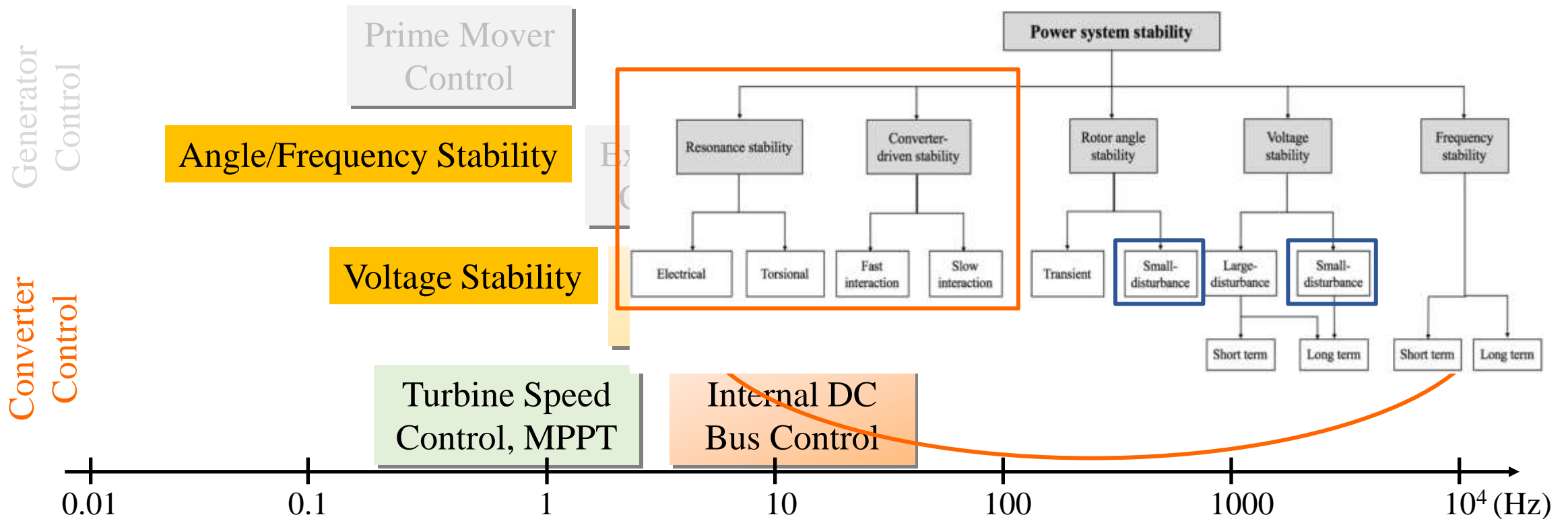
- Immittance is a Type of Transfer Functions (Frequency-Domain Models)
- Frequency-Domain Methods have Many Applications and Benefits
- Immittance-Based Stability Analysis has been Used in DC Power Systems
- Why not Doing More with it in AC Power Systems?
  - Practical Needs?
  - Theories and Analytical Development to Enable it
- Linearization of EMT Models Leads to Linear **Time-Periodic** Models

$$[x(\omega_1 t) + \Delta x] \cdot [y(\omega_1 t) + \Delta y] \rightarrow x(\omega_1 t)y(\omega_1 t) + y(\omega_1 t) \cdot \Delta x + x(\omega_1 t) \cdot \Delta y$$

J. Sun, “Small-signal methods for ac distributed power systems – a review,” in *Proc. COMPEL 2008*, also in *IEEE Trans. Power Electron.*, vol. 24, no. 11, pp. 2545-2554, November 2009.

# Need for New Stability Theory

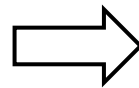
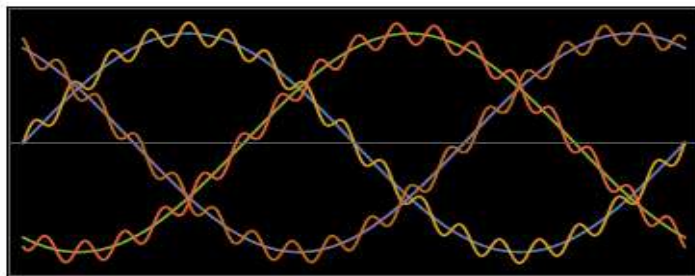
- Converters Introduce New Stability Problems in EMT Frequency Range
- Traditional Stability Study Based on RMS-Value Models is Insufficient



# Linearization and Frequency Scan

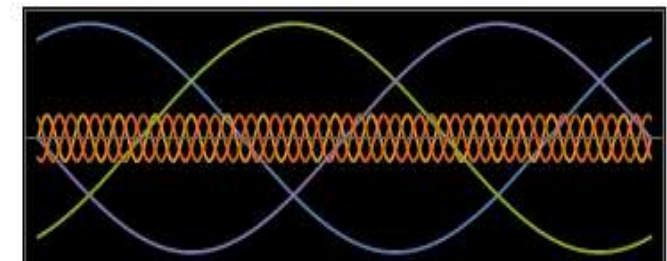
Perturbation	Domain	Perturbation
Amplitude Modulation	Time	$[V_r + \hat{v}_c(t)] \cos \omega_1 t - [V_i + \hat{v}_s(t)] \sin \omega_1 t$
	Frequency	$[V_r + \hat{V}_c \cos(\omega_p t + \varphi_c)] \cos \omega_1 t - [V_i + \hat{V}_s \cos(\omega_p t + \varphi_s)] \sin \omega_1 t$
Superimposed Perturbation	Time	$V_1 \cos(\omega_1 t + \varphi_1) + \hat{v}(t)$
	Frequency	$V_1 \cos(\omega_1 t + \varphi_1) + \hat{V}_p \cos(\omega_p t + \varphi_p)$

Compatible with Definition of Immittance



Converter to be Modeled

→  $f_1$   
 →  $f_p$   
 → Other  $f$



- Frequency Scan is Linearization in Frequency Domain – Harmonic Linearization
- Can Analytical Models be Developed Based on the Same Principle?

# Analytical Models

- The Answer is Yes, Although Hard

$$Y_{ap}(s) = \frac{1 - \frac{G_\theta(s - j\omega_1)}{2} [1 + \underline{Y}_1 H_{c0}(s - j\omega_1)]}{sL + R_L + H_{c0}(s - j\omega_1) - jK_{d0}} - \frac{j}{2} \cdot \frac{H_{va}(s - j\omega_1) H_{c0}(s - j\omega_1)}{sL + H_{c0}(s - j\omega_1) - jK_{d0}}$$

$s = j\omega$

- But There Are Complexities not Considered in Frequency Scan
  - Transfer Functions with Complex Coefficients
  - Need to Model in Positive and Negative Sequence
  - Coupling over Frequency, AC-DC Coupling



# Three Phase and Sequence Immittance

- Characterization of 3 $\Phi$  Converters Requires Independent 3 $\Phi$  Perturbations
- 3 $\Phi$  Variables can be Decomposed into Positive, Negative & Zero Sequence
  - Total Response is the Sum of Response to Perturbation in Each Sequence
  - Modeling can be Performed by Separate Perturbation in Each Sequence
  - Without Neutral Connection, Zero Sequence is an Open-Circuit
- Modeling by Positive- and Negative-Sequence Immittance

$$\mathbf{a} = e^{j\frac{2\pi}{3}}$$

$$\mathbf{S} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a} & \mathbf{a}^2 \\ 1 & \mathbf{a}^2 & \mathbf{a} \end{bmatrix}$$

Positive Sequence:

$$\begin{bmatrix} \hat{v}_a \\ \hat{v}_b \\ \hat{v}_c \end{bmatrix} = \hat{V}_p \begin{bmatrix} \cos(\omega_p t + \varphi_p) \\ \cos(\omega_p t + \varphi_p - 2\pi/3) \\ \cos(\omega_p t + \varphi_p - 4\pi/3) \end{bmatrix}$$

Negative Sequence:

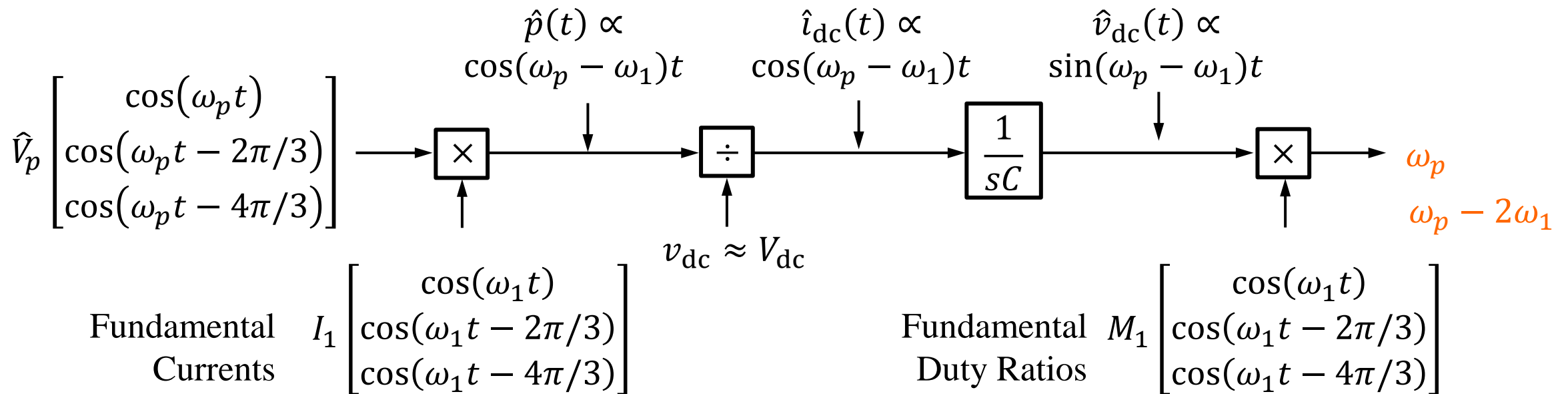
$$\hat{V}_p \begin{bmatrix} \cos(\omega_p t + \varphi_p) \\ \cos(\omega_p t + \varphi_p - 4\pi/3) \\ \cos(\omega_p t + \varphi_p - 2\pi/3) \end{bmatrix}$$

Zero Sequence:

$$\hat{V}_p \begin{bmatrix} \cos(\omega_p t + \varphi_p) \\ \cos(\omega_p t + \varphi_p) \\ \cos(\omega_p t + \varphi_p) \end{bmatrix}$$

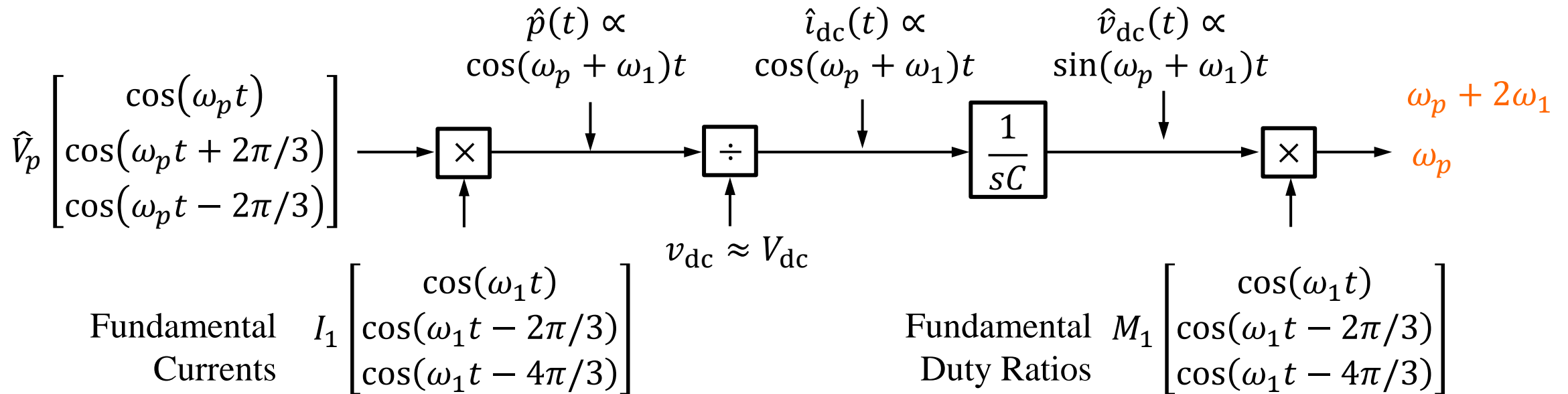
# Coupling over Frequency

- Positive-Sequence Voltage Perturbations at Frequency  $\omega_p$  Produces
  - DC-Port Power and DC Bus Voltage Perturbation at  $\omega_p - \omega_1$
  - Positive-Sequence Voltage and Current at Converter Terminal at  $\omega_p$
  - Negative-Sequence Voltage and Current at Converter Terminal at  $\omega_p - 2\omega_1$



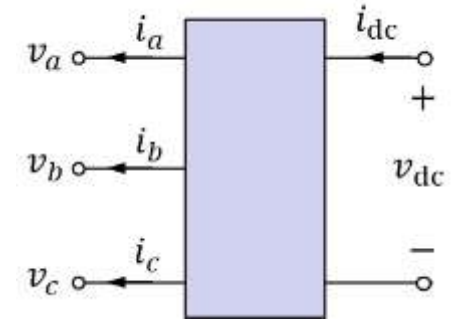
# Coupling over Frequency – Cont'd

- Negative-Sequence Voltage Perturbations at Frequency  $\omega_p$  Produces
  - Power and DC Bus Voltage Perturbation at  $\omega_p + \omega_1$
  - Negative-Sequence Voltage and Current at Converter Terminal at  $\omega_p$
  - Positive-Sequence Voltage and Current at Converter Terminal at  $\omega_p + 2\omega_1$



# Two-Port Modeling

- Three-Phase VSC as Building Blocks for Grid Application
- Characterization as Two-Port Network
- Full Model Involves 9 Transfer Functions (Admittances)



Voltage Perturbation	Self Admittance	Transfer Admittance	
AC Port, Positive Sequence $\hat{v}_p(s)$	$Y_{pp}(s) = -\frac{\hat{i}_a(s)}{\hat{v}_p(s)}$	$Y_{pd}(s) = -\frac{\hat{i}_{dc}(s - j\omega_1)}{\hat{v}_p(s)}$	$Y_{pn}(s) = -\frac{\hat{i}_a(s - j2\omega_1)}{\hat{v}_p(s)}$
AC Port, Negative Sequence $\hat{v}_n(s)$	$Y_{nn}(s) = -\frac{\hat{i}_a(s)}{\hat{v}_n(s)}$	$Y_{nd}(s) = -\frac{\hat{i}_{dc}(s + j\omega_1)}{\hat{v}_n(s)}$	$Y_{np}(s) = -\frac{\hat{i}_a(s + j2\omega_1)}{\hat{v}_n(s)}$
DC Port $\hat{v}_{dc}(s)$	$Y_{dd}(s) = \frac{\hat{i}_{dc}(s)}{\hat{v}_{dc}(s)}$	$Y_{dp}(s) = \frac{\hat{i}_a(s + j\omega_1)}{\hat{v}_{dc}(s)}$	$Y_{dn}(s) = \frac{\hat{i}_a(s - j\omega_1)}{\hat{v}_{dc}(s)}$

J. Sun, “Two-port characterization and transfer immittances of ac-dc converters – Part I & II, IEEE OJ-PEL, Aug. 2021.

# Change of Sequence and Frequency

- $x(t) = X \cos(\omega t + \varphi) \rightarrow X e^{j\varphi} e^{j\omega t} \rightarrow \underline{X} e^{j\omega t} \rightarrow \langle \omega, \underline{X}, \sigma \rangle, \quad \sigma = (\text{PS}, \text{NS})$
- Balanced 3 $\Phi$  in NS at Frequency  $\omega$  can be Expressed as PS at Frequency  $-\omega$

$$\begin{bmatrix} x \cos(\omega t + \varphi) \\ x \cos\left(\omega t + \varphi + \frac{2\pi}{3}\right) \\ x \cos\left(\omega t + \varphi - \frac{2\pi}{3}\right) \end{bmatrix} = \begin{bmatrix} x \cos((-\omega)t - \varphi) \\ x \cos\left((-\omega)t - \varphi - \frac{2\pi}{3}\right) \\ x \cos\left((-\omega)t - \varphi + \frac{2\pi}{3}\right) \end{bmatrix}$$

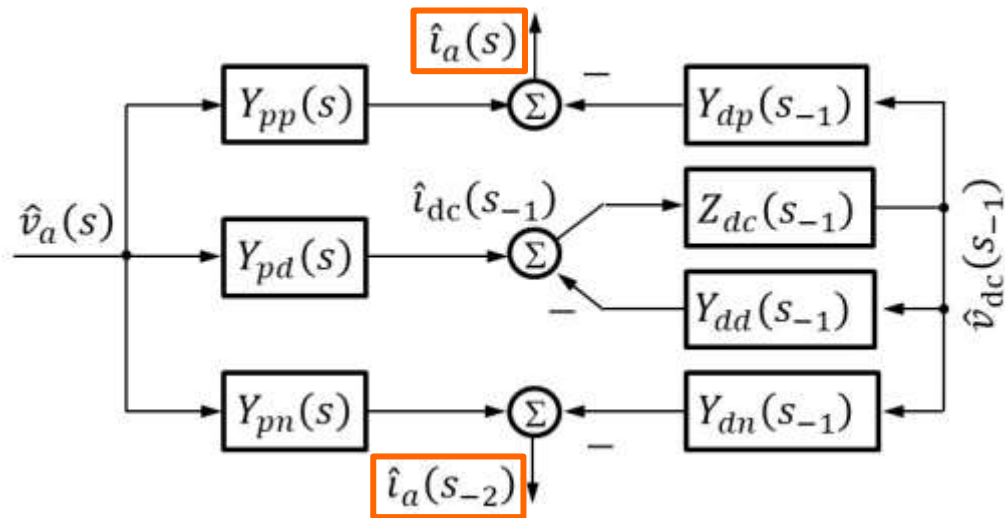
$\langle \omega, \underline{X}, \sigma \rangle = \langle -\omega, \underline{X}^*, \bar{\sigma} \rangle$

$\overline{\text{PS}} = \text{NS}, \quad \overline{\text{NS}} = \text{PS}$

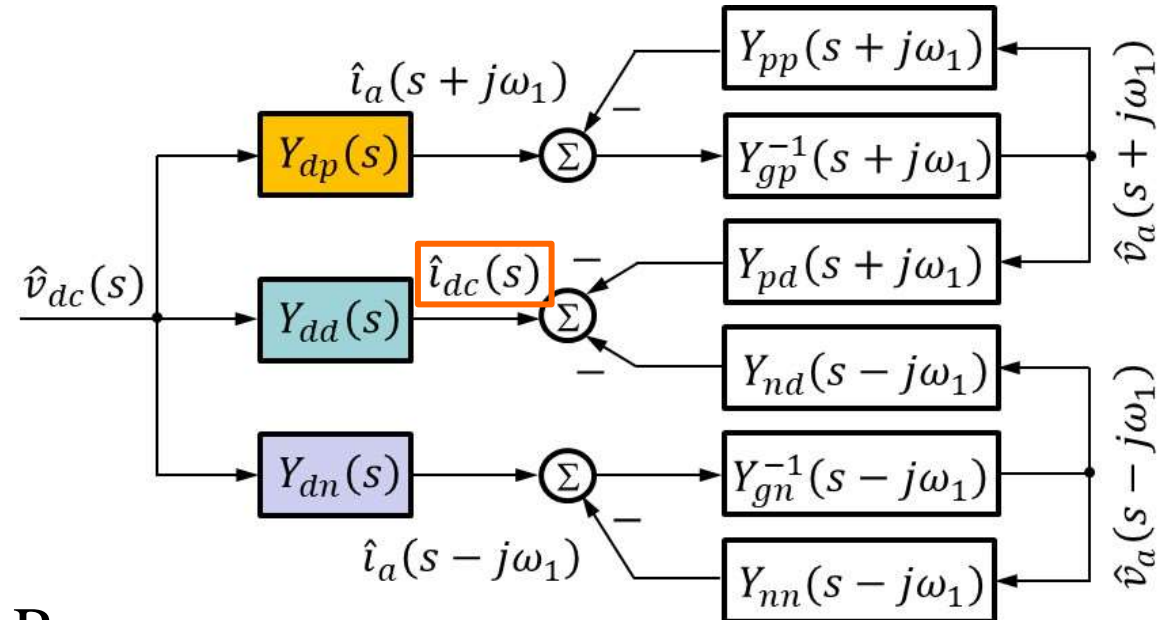
- Negative-Sequence Immittance at Frequency  $\omega =$  Complex Conjugate of Positive-Sequence Immittance at Frequency  $-\omega$ , Vice Versa
- Complete Characterization by 5 Transfer Functions  $\{Y_{pp}, Y_{pd}, Y_{pn}, Y_{dd}, Y_{dp}\}$

# Effects of AC-D Coupling

- $Z_{dc}$  Affects AC-Port Immittance



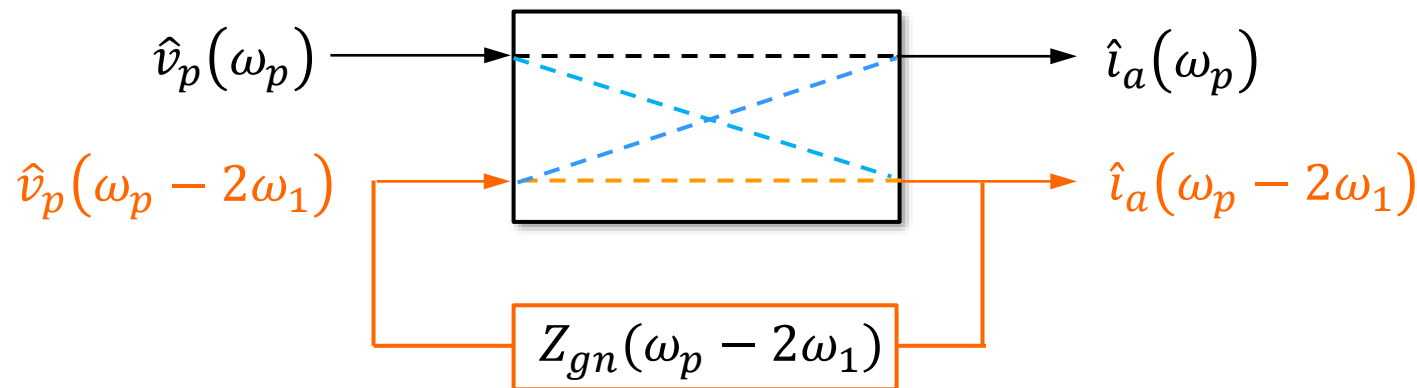
- $Z_{ac}$  Affects DC-Port Immittance



- Reflection Creates Additional Current Responses
  - Model by Equivalent Parallel Immittance or Direct Frequency Scan of Full Model
  - Two-Port Models Provide More Flexibility & Enable Analysis of AC-DC Grid

# Effects of Coupled Current at $\omega_p \pm 2\omega_1$

- With a Positive-Sequence Voltage Perturbation at  $\omega_p$ 
  - Transfer Admittance  $Y_{cp}(j\omega_p)$  Creates a Current Response at  $\omega_p - 2\omega_1$
  - Coupled Current Induces Voltage at  $\omega_p - 2\omega_1$  Through  $Y_{gn}(j\omega_p - j2\omega_1)$
  - Induced Voltage Adds a Current Response at  $\omega_p$  Through  $Y_{cn}(j\omega_p - j2\omega_1)$
  - Net Effects can be Modeled by an Equivalent Parallel Admittance  $Y_{ep}$



$$Y_{ep}(s) = -\frac{Y_{cn}(s_{-2})Y_{cp}(s)}{Y_{an}(s_{-2}) + Y_{gn}(s_{-2})}$$

$$Y_{en}(s) = -\frac{Y_{cp}(s_{+2})Y_{cn}(s)}{Y_{ap}(s_{+2}) + Y_{gp}(s_{+2})}$$

# Immittance-Based Stability Theory

- Characterize Each *Converter* by Its Small-Signal Immittances
  - Analytical Modeling, Frequency Scan, Physical Measurement
  - Standalone Operation, Ideal External Conditions
- Formulate System Models by Linear Network Analysis
- Reformulate System Models to Resemble a Feedback Loop
  - With Open-Loop Stability Guaranteed by Practical Considerations
- Apply Nyquist Criteria to Assess System Stability
- Repeat Analysis in Positive and Negative Sequence

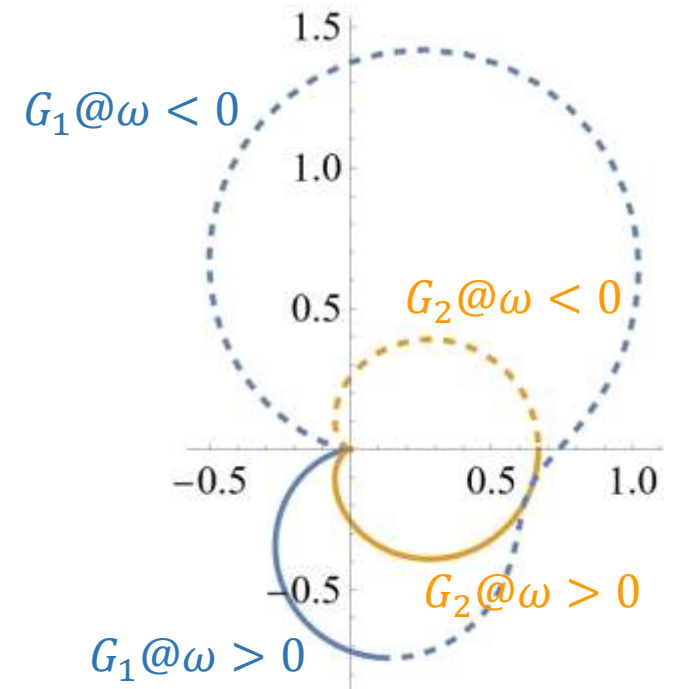


# Why Positive AND Negative Sequence?

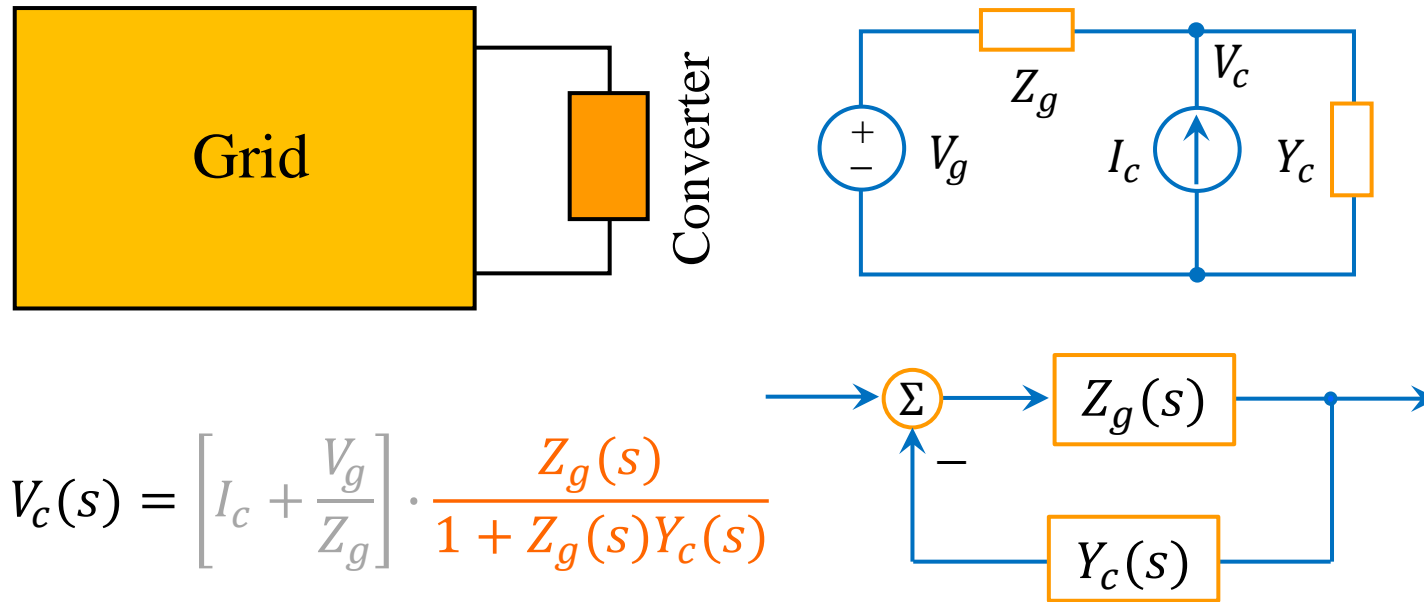
- Instability May Occur in Either Sequence
  - Especially with Control in Both Sequences
- Immittances Models have Complex Coefficients
  - Frequency Response May not be Complex-Conjugate Symmetric about  $\omega = 0$
  - Nyquist Analysis Must Cover Positive & Negative  $\omega$
  - Response at Negative Frequency cannot be Scanned
  - (PS Model at  $-\omega$ )<sup>\*</sup> = NS Model at  $-\omega$
  - PS Response at  $-\omega = (\text{NS Response at } +\omega)$ <sup>\*</sup>
- System Stability = Stability in PS & NS for  $\omega > 0$

$$G_1(s) = \frac{12}{(s+2)(s+1+j8)}$$

$$G_2(s) = \frac{12}{(s+2)(s+9)}$$



# Stability at Converter-Grid Interface



IEEE TRANSACTIONS ON POWER ELECTRONICS, VOL. 26, NO. 11, NOVEMBER 2011

## Letters

### Impedance-Based Stability Criterion for Grid-Connected Inverters

Fan Sun

**Abstract**—Grid-connected inverters are known to become unstable when the grid impedance is high. Existing approaches to analyzing such instability are based on inverter control models that account for the grid impedance and the coupling with other grid-connected inverters. A new method to determine inverter-grid system stability using only the inverter output impedance and the grid impedance is developed in this paper. It will be shown that a grid-connected inverter will remain stable if the ratio between the grid impedance and the inverter output impedance satisfies the Nyquist stability criterion. This new impedance-based stability criterion is a generalization to the existing stability criterion for voltage-source systems, and can be applied to all current-source systems. A single-phase solar inverter is studied to demonstrate the application of the proposed method.

**Index Terms**—Current source systems, grid-connected inverters, harmonic resonance, impedance analysis, small-signal stability.

#### 1. INTRODUCTION

**M**OST renewable energy sources are connected to the power grid through an inverter. In such grid-connected mode, the inverter is typically controlled as a current source [1] to inject certain amount of current into the grid. Dynamic interaction of grid-connected inverters with the power grid has been a topic of extensive study in recent years due to the rapidly increasing penetration of renewable energy and distributed generation (DG) resources. Traditional power system theory uses phasor-based models to study the effects of DG sources on grid

pling with other grid-connected inverters, and use root locus and other time- or frequency-domain techniques to determine inverter control stability under variable grid conditions.

Inverter instability in the presence of high grid impedance is similar to the converter-filter or the more general source-load interaction problems found in many other power electronic systems. A well-established technique to analyze such interconnected systems is by the impedance-based stability criterion: The ratio of the source output impedance to the load input impedance must satisfy the Nyquist stability criterion in order for the interconnected source-load system to be stable. The technique is widely used in the design of switching-mode power supplies with input filters [8], as well as more complex ac distributed power systems [9]. Generalization of the technique to ac power systems has also been reported [10].

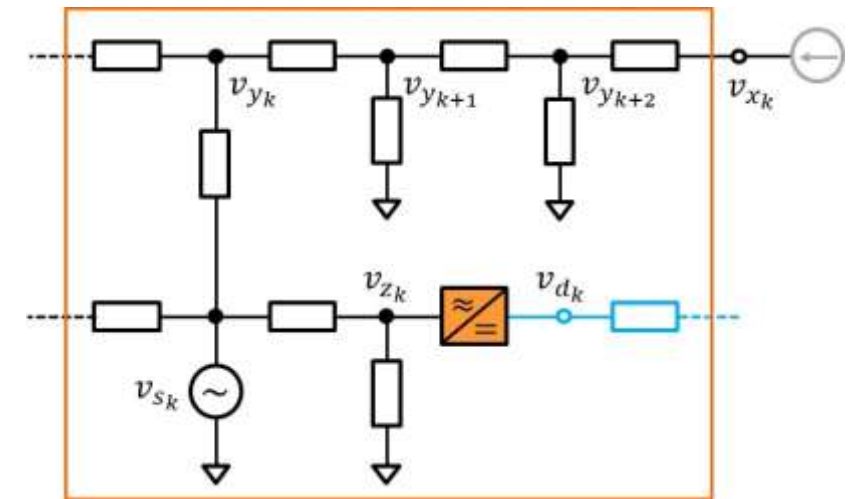
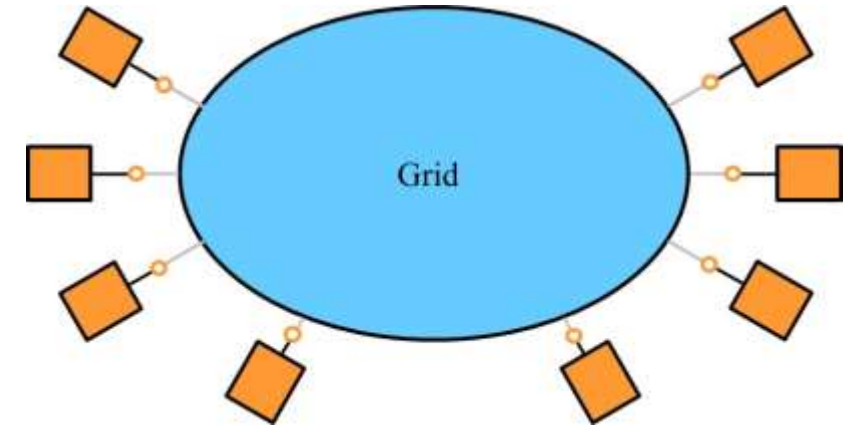
Detailed inverter control models and their loop stability analysis are necessary for the design of individual grid-connected inverters. When the objective is grid system stability analysis, external behavior of an inverter is of more interest than its internal loop stability and is also easier to obtain. In such cases, the impedance-based approach is more advantageous and effective [10], as it avoids the need to remodel each inverter and repeat its loop stability analysis when the grid impedance changes, or when more inverters are connected to the same grid. It also does not require detailed design information of individual inverters, which is often not available in those performing grid system stability analysis.

- Converter-Grid System Stability = Stability of a SISO Feedback Loop
- Practical Considerations Guarantee Open-Loop Stability
- Nyquist Criterion without the Need to Check Open-Loop Right Half Plane Poles

# Multi-Converter Systems

- Replace Each Converter by Its Admittances
  - Power System → Linear Network
  - $n$  Converters Nodes,  $m(> n)$  Total Nodes
- Consider Disturbance in Node Currents
- Build a System Model in the Frequency-Domain by Nodal Analysis

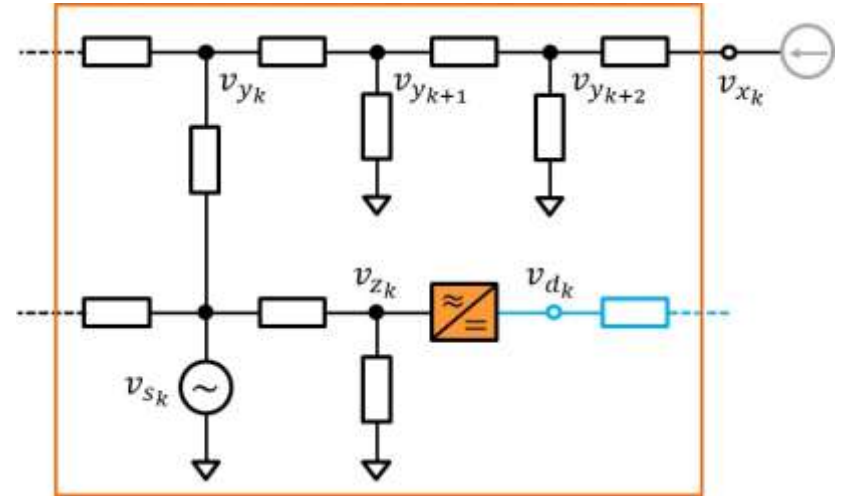
$$\begin{bmatrix} y_{11}(s) & y_{12}(s) & \cdots & y_{1m}(s) \\ y_{21}(s) & y_{22}(s) & \cdots & y_{2m}(s) \\ \vdots & \vdots & \ddots & \vdots \\ y_{m1}(s) & y_{m2}(s) & \cdots & y_{mm}(s) \end{bmatrix} \begin{bmatrix} v_1(s) \\ v_2(s) \\ \vdots \\ v_m(s) \end{bmatrix} = \begin{bmatrix} i_1(s) \\ i_2(s) \\ \vdots \\ i_m(s) \end{bmatrix}$$



# Stability Analysis by Direct Methods

$$\begin{bmatrix} y_{11}(s) & y_{12}(s) & \cdots & y_{1m}(s) \\ y_{21}(s) & y_{22}(s) & \cdots & y_{2m}(s) \\ \vdots & \vdots & \ddots & \vdots \\ y_{m1}(s) & y_{m2}(s) & \cdots & y_{mm}(s) \end{bmatrix} \begin{bmatrix} v_1(s) \\ v_2(s) \\ \vdots \\ v_m(s) \end{bmatrix} = \begin{bmatrix} i_1(s) \\ i_2(s) \\ \vdots \\ i_m(s) \end{bmatrix}$$

$$\mathbf{v}(s) = \mathbf{Y}^{-1}(s)\mathbf{i}(s) = \mathbf{Z}(s)\mathbf{i}(s) \quad \mathbf{Z}(s) \triangleq \mathbf{Y}^{-1}(s)$$



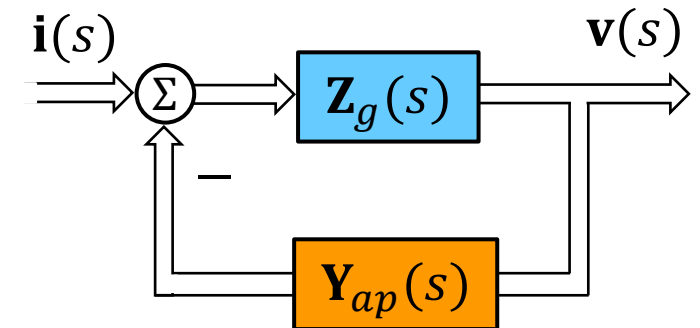
- System is Stable  $\Leftrightarrow \mathbf{Z}(s)$  is Stable
- $\mathbf{Z}(s)$  is Stable if it has no Right-Half Plane (RHP) Poles (Matrix)
- Numerical Methods Required to **Invert  $\mathbf{Y}(s)$**  and **Determine Poles of  $\mathbf{Z}(s)$** 
  - Difficult for Large Systems; Especially When Using Frequency Scan Data
  - Lack Insights; Gives no Indication for Source of Instability or Possible Solutions

# Reformulating System Models

- Separation of Nodal Admittance Matrix:  $\mathbf{Y}(s) = \mathbf{Y}_N(s) + \mathbf{Y}_{ap}(s)$ 
  - $\mathbf{Y}_N(s)$ : Network without Added Converters (but May Include Other Converters)
  - $\mathbf{Y}_{ap}(s)$ : Converter Admittances

$$\mathbf{v}(s) = \mathbf{Y}^{-1}(s)\mathbf{i}(s) = [\mathbf{Y}_N(s) + \mathbf{Y}_{ap}(s)]^{-1}\mathbf{i}(s)$$

$$= [\mathbf{I} + \mathbf{Z}_g(s)\mathbf{Y}_{ap}(s)]^{-1}\mathbf{Z}_g(s)\mathbf{i}(s) \quad \mathbf{Z}_g(s) = \mathbf{Y}_N^{-1}(s)$$



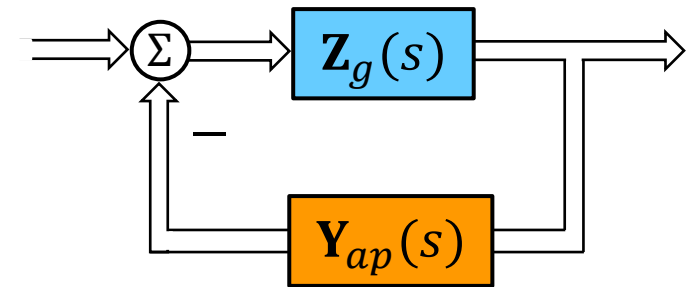
- Operation without Converters:  $\mathbf{v}_0(s) = \mathbf{Z}_g(s)\mathbf{i}(s)$ 
  - Grid and Converters are Stable  $\rightarrow \mathbf{Z}_g(s)$  and  $\mathbf{Y}_{ap}(s)$  have no RHP Poles
- Interconnected System Stability can be Determined by Applying the Generalized Nyquist Criterion to  $\mathbf{Z}_g(s)\mathbf{Y}_{ap}(s)$

# Other Complexities and Extensions

- Coupling over Frequency and AC-DC Coupling

$$\mathbf{Y}_{ap}(s) \rightarrow \mathbf{Y}_{ap}(s) - \mathbf{Y}_{cn}(s_{-2})[\mathbf{Y}_{an}(s_{-2}) + \mathbf{Y}_{gn}(s_{-2})]^{-1}\mathbf{Y}_{cp}(s)$$

- Grid Following vs. Grid Forming Converters
- AC, DC and Hybrid AC-DC Grids
- Addition, Replacement or Removal of Converters
- Same Modeling Procedure; Final Models in Similar Form
  - Feedback Loop with Guaranteed Open-Loop Stability
- NS Model can be Obtained by Complex Conjugate of PS Model at  $-\omega$ 
  - Stability Must be Checked Separately



J. Sun, "Frequency-Domain Stability Criteria for Converter-Based Power Systems," in *IEEE Open J. Power Electronics*, vol. 3, pp. 222-254, 2022

# Summary

- Analytical Development and Models Revealed
  - Complex Coefficients of Immittance Models as Transfer Functions
  - Need to Model in Both Positive and Negative Sequence
  - Coupling over Frequency, AC-DC Coupling; System Effects
- Immittance-Based Frequency-Domain Stability Theory
  - General Formulation of System Models; Feedback Form to Simplify Application
- Frequency Scan is a Simplified Special Application
  - Positive Sequence at Positive Frequency, SISO Model
  - Negative Sequence Should be Included for Completeness
  - Coupling over Frequency Should be Added for Accuracy

# Outline

- Theory
- Applications

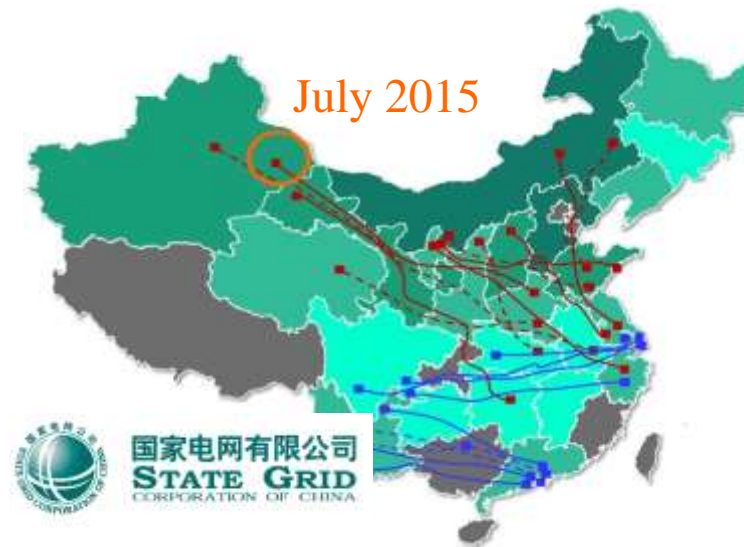


# Scope of Applications

- Types of Components
  - Wind Turbines, PV Inverters, Energy Storage, Load Converters
  - HVDC Converters (LCC, VSC, MMC), STATCOM, Solid-State Transformers
  - Synchronous Generators, Synchronous Condensers, Induction Machines
- Types of Systems
  - Renewable Generation, HVDC Transmission, Distribution, and Load Centers
  - Plant-Grid, Intra-Plant, Inter-Plant, and Overall Grid System Stability
  - AC, DC, and Hybrid AC-DC Grids; Microgrids
- Types of Stability Problems
  - Subsynchronous X, Supersynchronous Resonance, High-Frequency Resonance

# Test by Real-World Problems

- First of Their Kinds; Success Promoted Early Adoption
  - Offshore Wind Farms with HVDC Transmission (~290 Hz Resonance)
  - Onshore Wind, Thermal Power Plants and HVDC (70-80 Hz Resonance)
  - Data Center Power Systems (~70 Hz and HF Resonance)



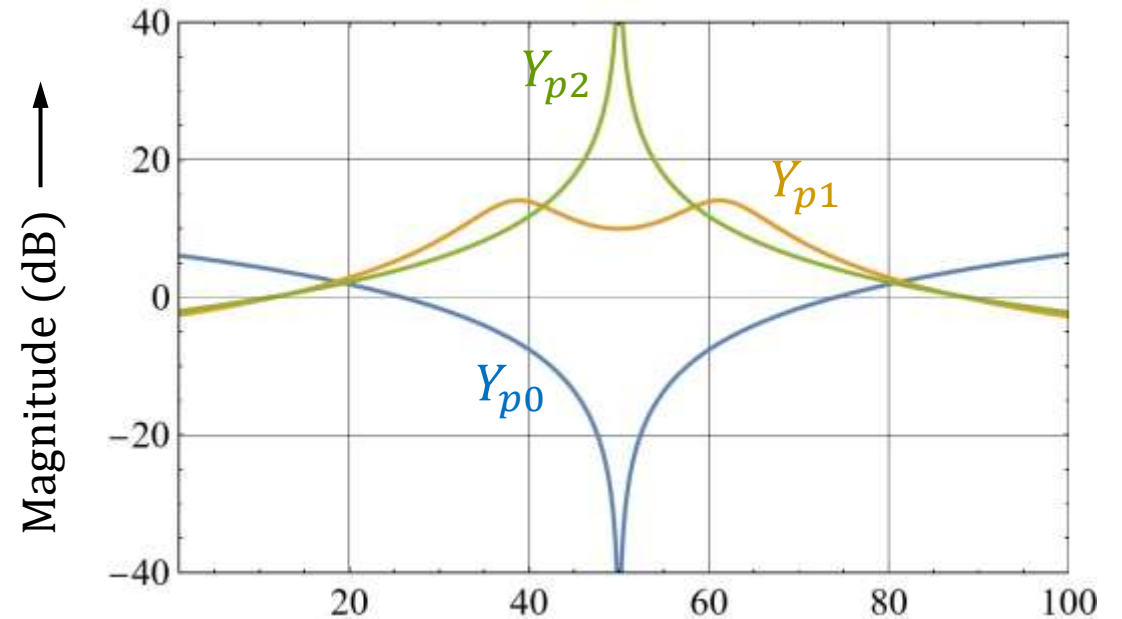
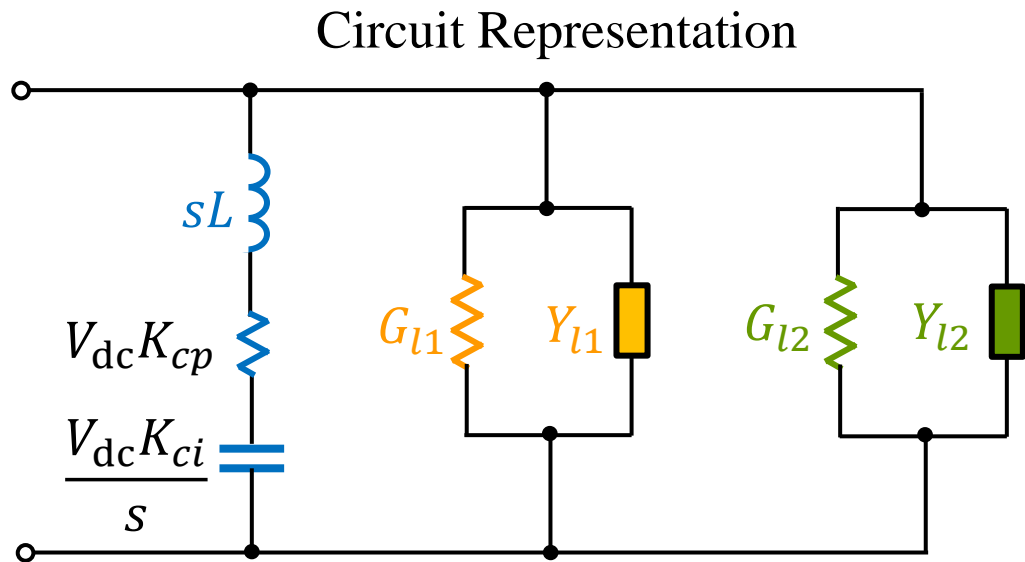
# Beyond Stability Analysis

- Common Modes and Root Causes of Resonance and Instability
  - Formation of Resonance, Inductive vs. Capacitive Impedance
  - Source of Negative Damping
- Mitigation Methods; Design for Stability
  - Control Tuning, Active Damping, etc. to Solve Specific Problems
  - General Design Methodology to Prevent Common Problems
  - Product Specifications, Standards to Guarantee Stability
- Enabled by Insights Gained from Analytical Models
  - Effects of Different Control Function on Immittance
  - Characteristics of Immittance Responses in Different Frequency Ranges

# Breakdown of VSC Admittance

- Each Control Function is Equivalent to a Parallel Impedance
  - Current Control, PLL, AC Voltage Control

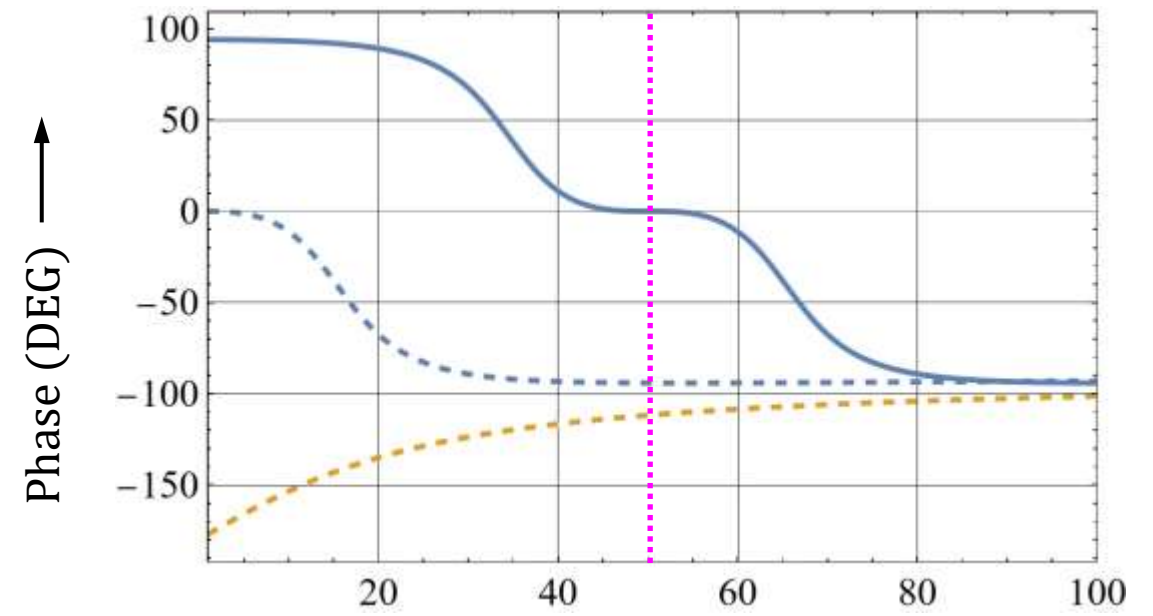
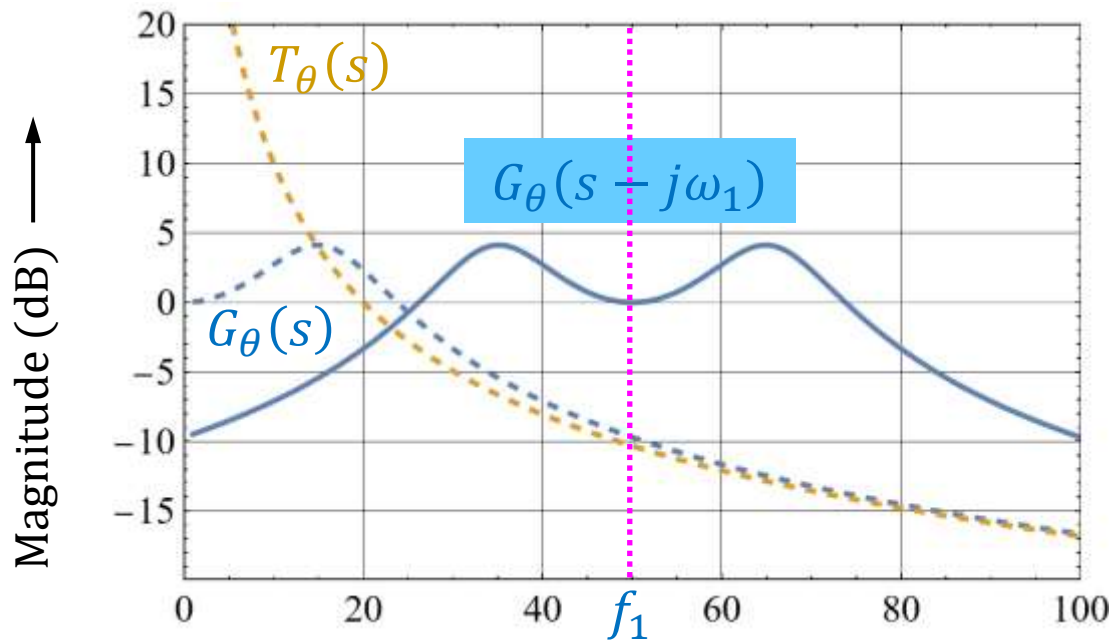
$$Y_{ap}(s) \approx \frac{1}{sL + H_{c0}(s - j\omega_1)} - \frac{\underline{Y}_1 G_\theta (s - j\omega_1)}{2} - \frac{jH_{va}(s - j\omega_1)}{2} \triangleq Y_{p0}(s) + Y_{p1}(s) + Y_{p2}(s)$$



# Effects of PLL

- $T_\theta(s)$  is PLL Loop Gain,  $G_\theta(s)$  is Closed-Loop Transfer Function
- $|G_\theta(s - j\omega_1)|$  is Symmetrical about the Fundamental
  - Phase Reversal about  $f_1$  but Stays within  $\pm 90^\circ$

$$Y_{p1}(s) = -\frac{Y_1 G_\theta(s - j\omega_1)}{2}$$

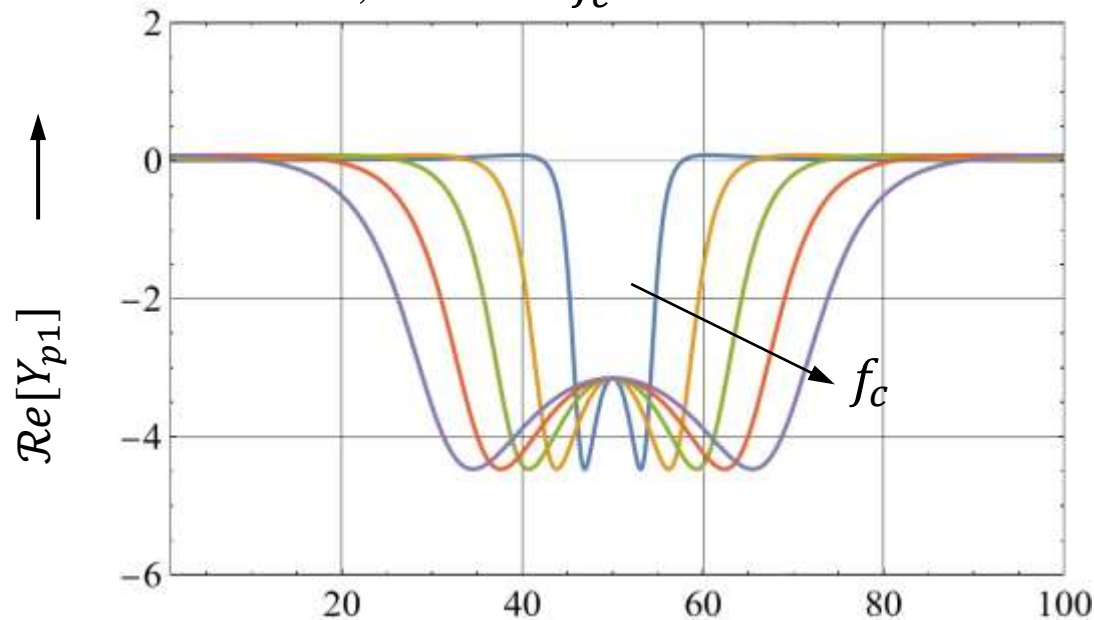


# Inversion Mode, Unity Power Factor

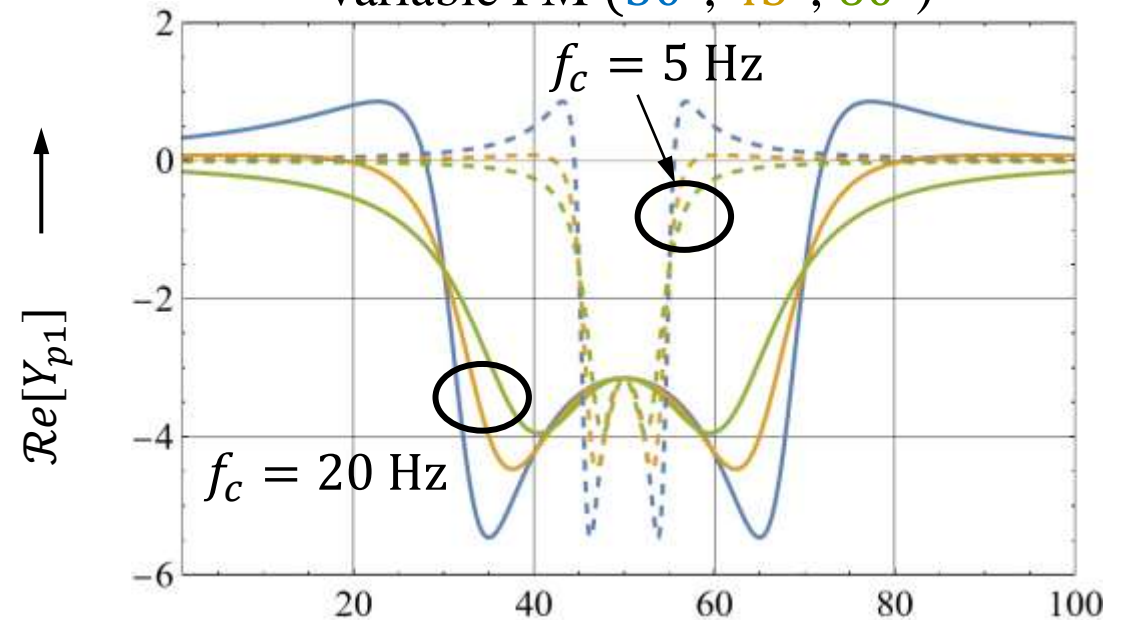
- $\underline{Y}_1$  (Admittance at  $\omega_1$ ) is Positive Real
- Positive Real Part of  $G_\theta(s - j\omega_1) \rightarrow$  Negative Damping
  - Amount and Frequency Range Depends on PLL Design

$$Y_{p1}(s) = -\frac{\underline{Y}_1 G_\theta(s - j\omega_1)}{2}$$

45° PM, Variable  $f_c$  from 5 Hz to 25 Hz



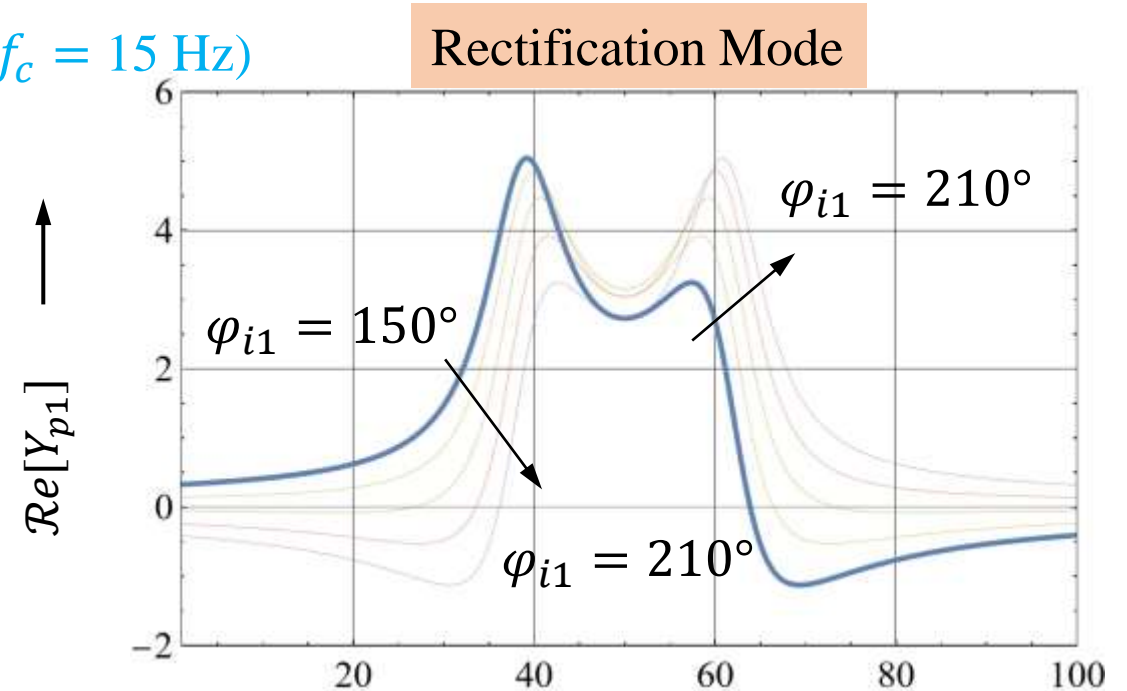
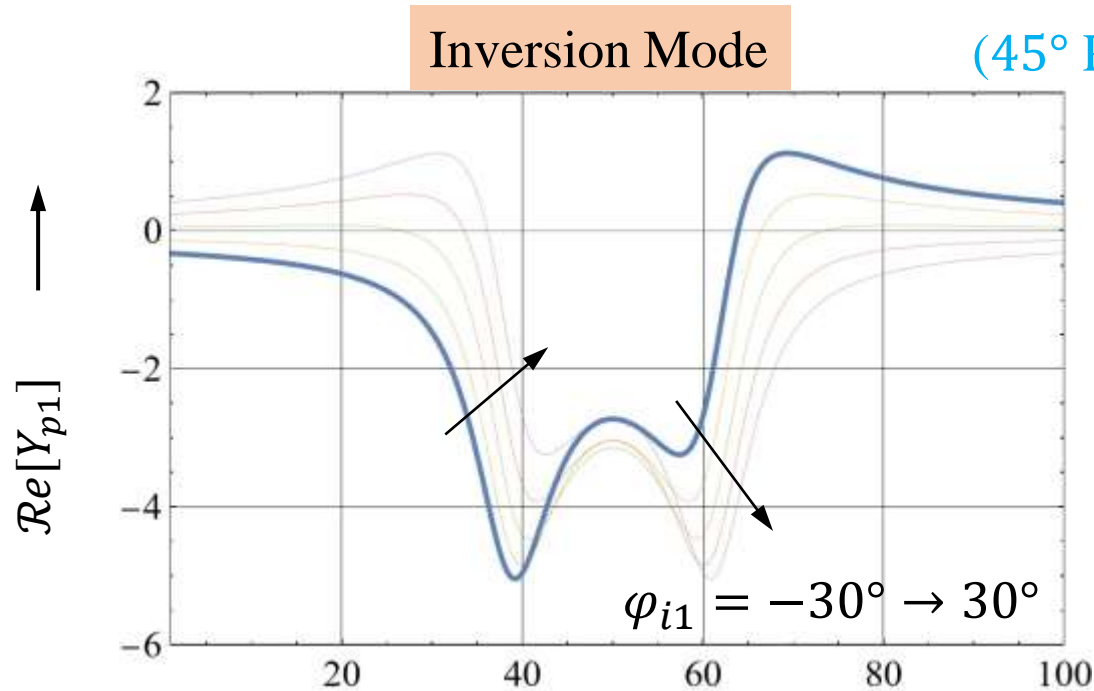
Variable PM (30°, 45°, 60°)



# Effects of Power and Power Factor

- Amount of Negative Damping is Proportional to Power
- Positive Damping in Rectification Mode
- Non-Unity Power Factor Causes Asymmetry About  $f_1$

$$Y_{p1}(s) = -\frac{Y_1 G_\theta (s - j\omega_1)}{2}$$

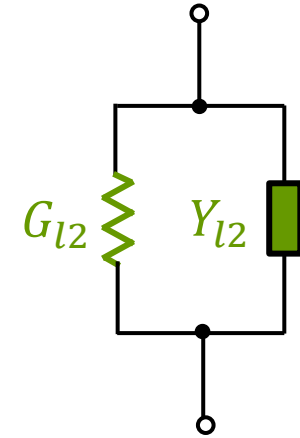


# Effects of AC Voltage Control

$$Y_{p2}(s) = -\frac{jH_{va}(s-1)}{2}$$

$$H_{va}(s) = K_{vp} + \frac{K_{vi}}{s}$$

$$Y_{p2}(j\omega) = \frac{1}{j2/K_{vp}} + \frac{1}{2} \frac{K_{vi}}{\omega_1 - \omega} \triangleq jY_{l2} + G_{l2}$$

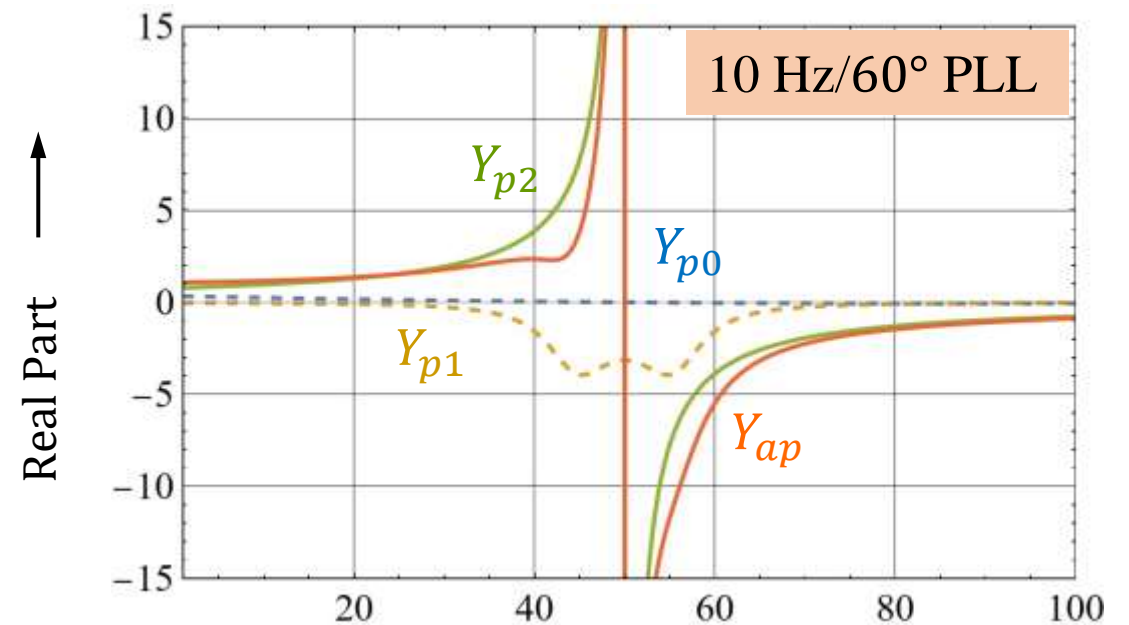
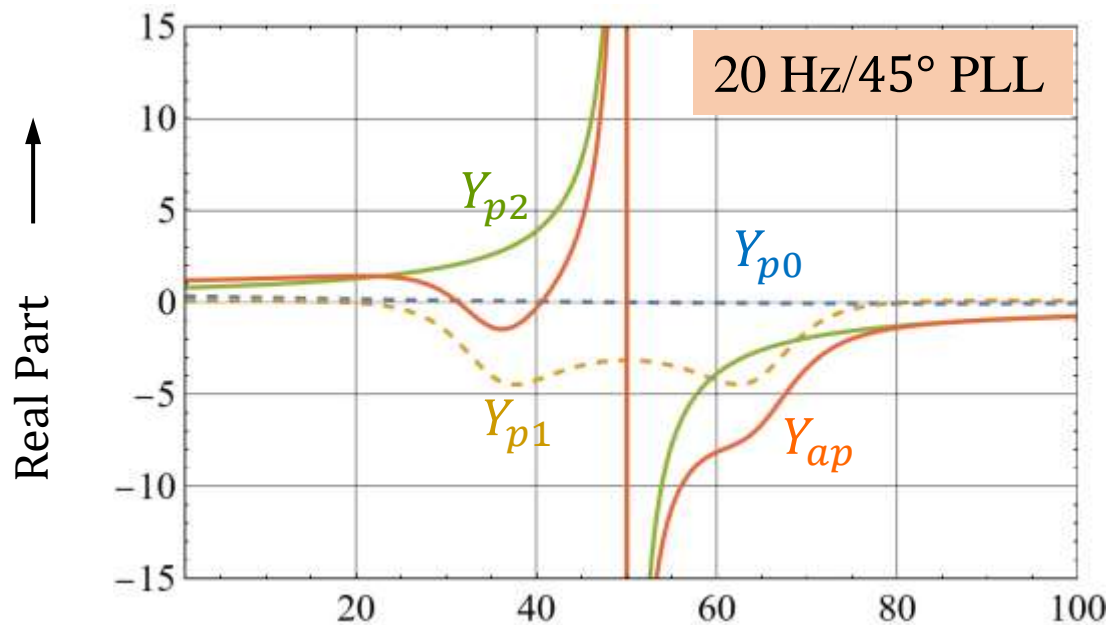


- Assume PI Compensator
- Proportional Gain Contributes a Constant Inductive Reactance
- Integral Gain is Equivalent to a Frequency-Dependent Resistor
  - Positive (Damping) Below  $f_1$ ; Negative Above  $f_1$
- Magnitude Response is Symmetrical About the Fundamental Frequency



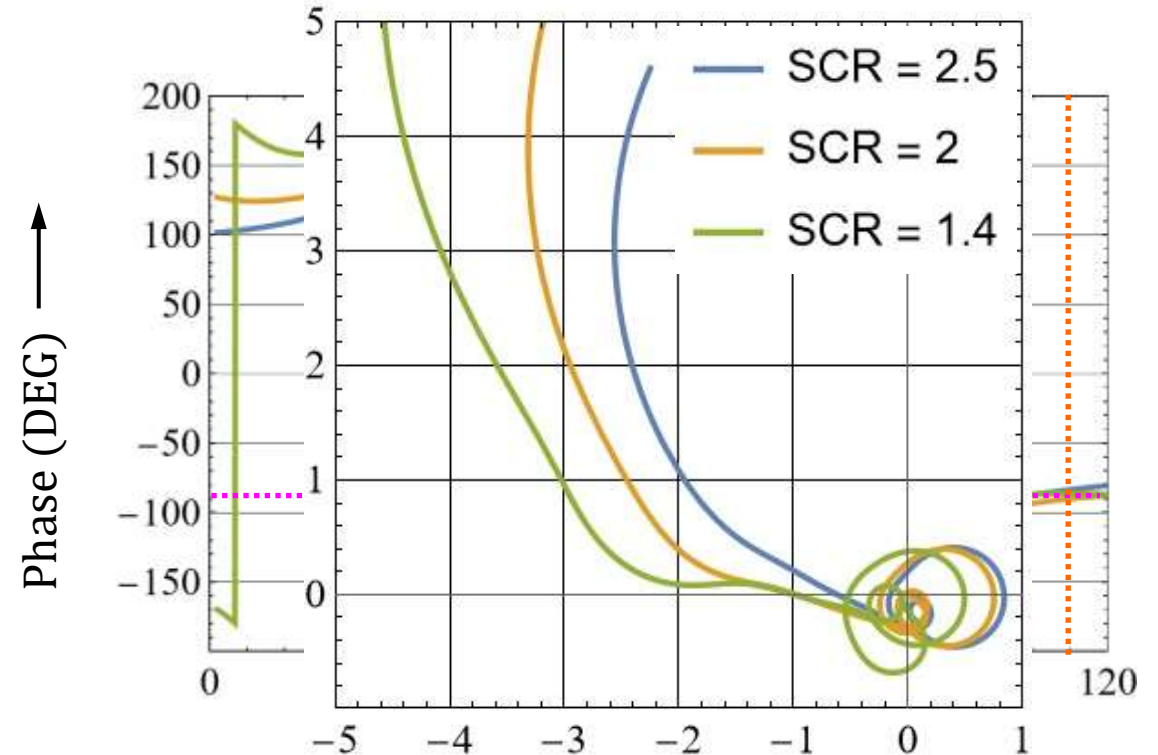
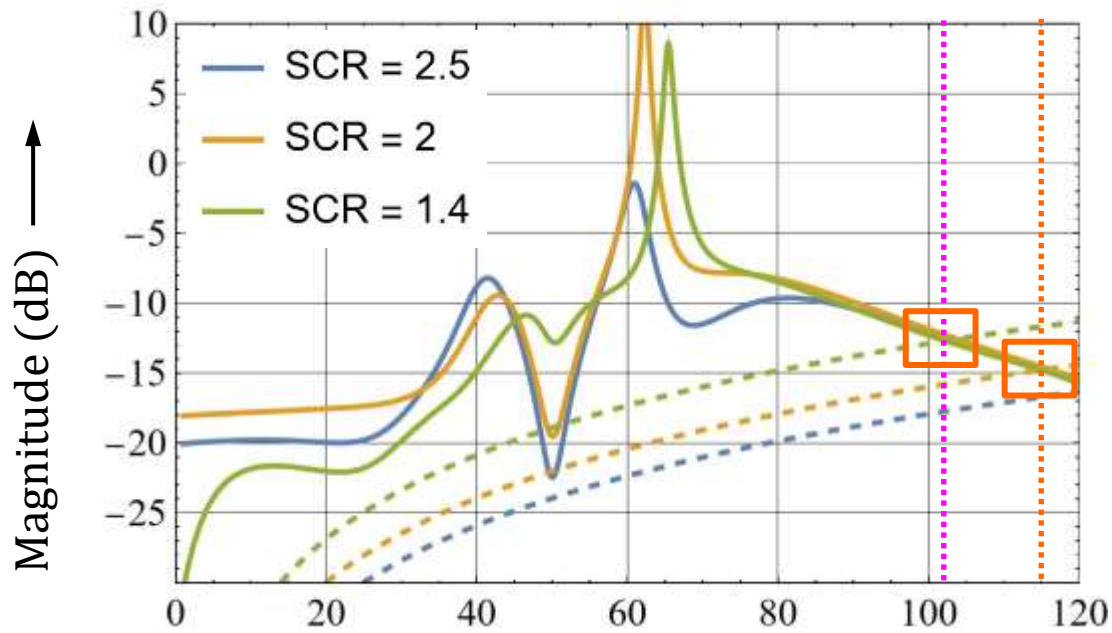
# Damping of Overall Immittance

- Positive Damping Below  $f_1$  can Cancel Negative Damping Caused by PLL
- Negative Damping Above  $f_1$  Adds to Negative Damping Caused by PLL
  - Negative Damping Above  $f_1$ , **Root Cause for Supersynchronous Resonance**



# Supersynchronous Resonance in Weak Grid

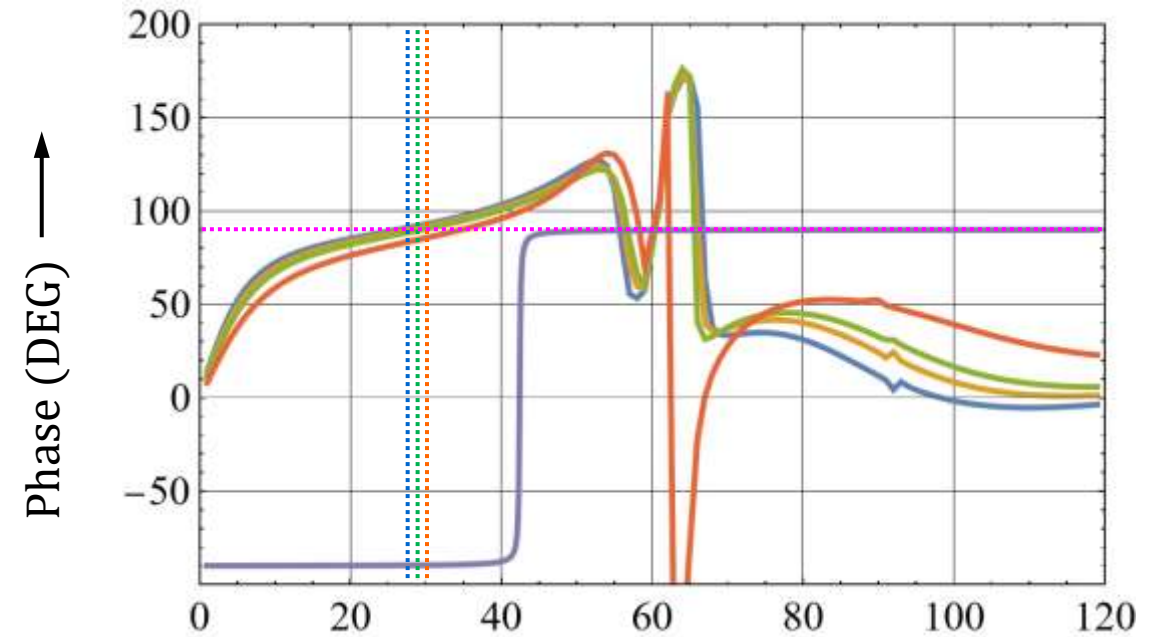
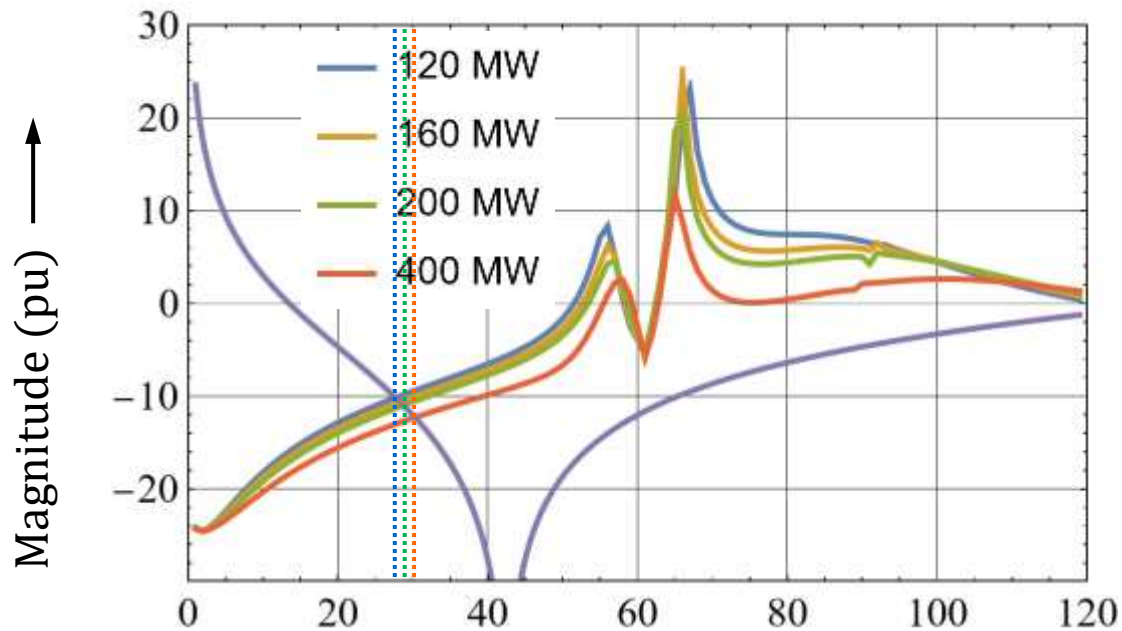
- Common Problem for PV Inverters and Type-IV Turbines in Weak Grids
  - Capacitive Impedance Dominated by Current Control; High Inductive Grid Impedance
  - Negative Damping Caused by PLL and AC Voltage Control



# Type-III Turbines and SSR

- Proportional Gain of RSC Current Control is Equivalent to a Resistor in Series with  $R_r$
- IG Effects Create Negative Damping below  $f_m$  → SSR with Series Compensated Line

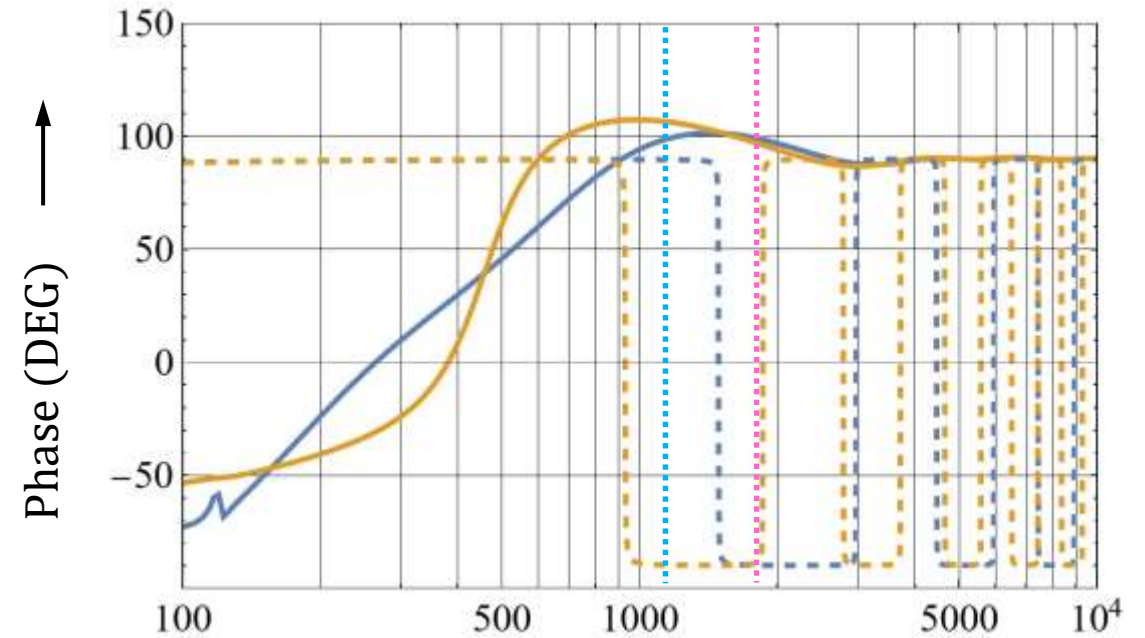
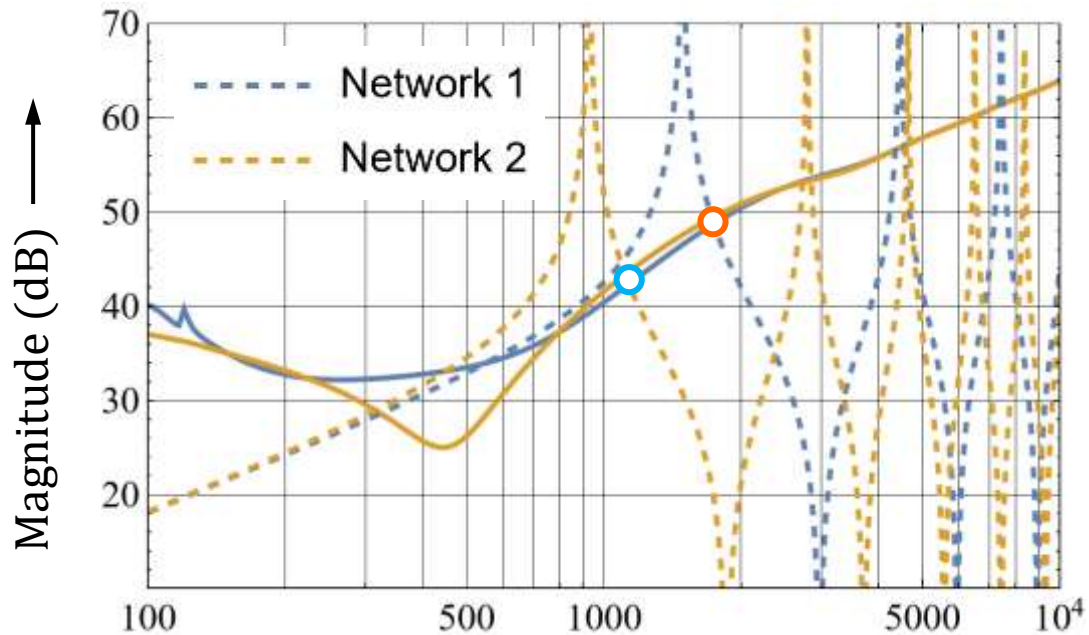
$$Z_{rsc}(s) = sL'_{lr} + \frac{1}{\alpha^2} \cdot \frac{s}{s - j\omega_m} \left[ R_r + K_{rpo} + \frac{K_{ri0}}{s - j\omega_1} - jK_{rd0} \right]$$



# High-Freq. Resonance of HVDC Converters

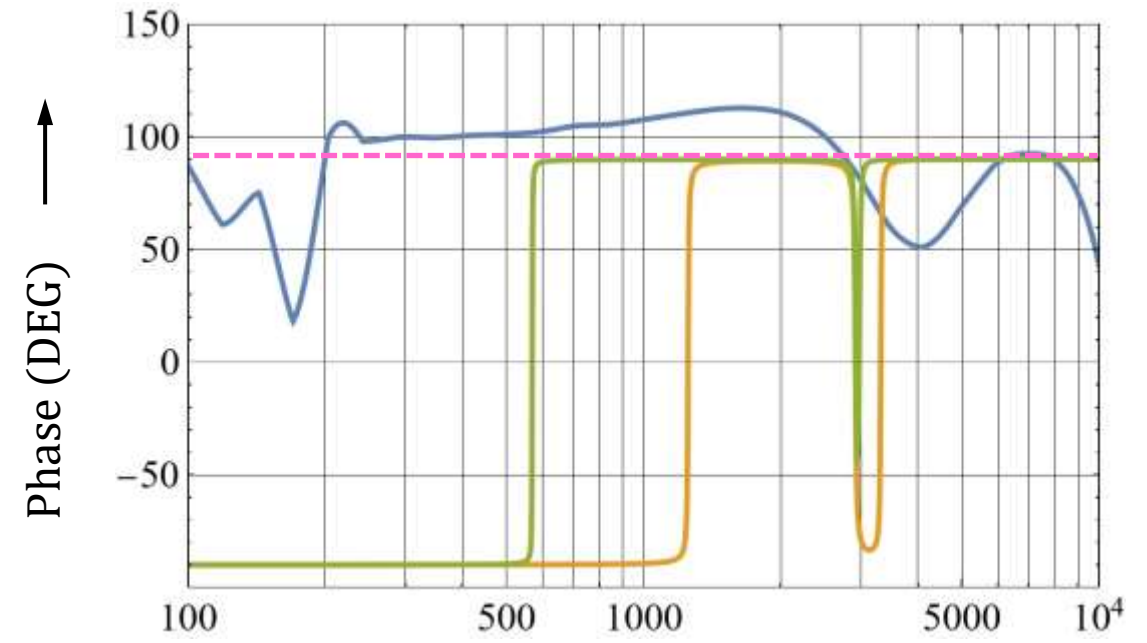
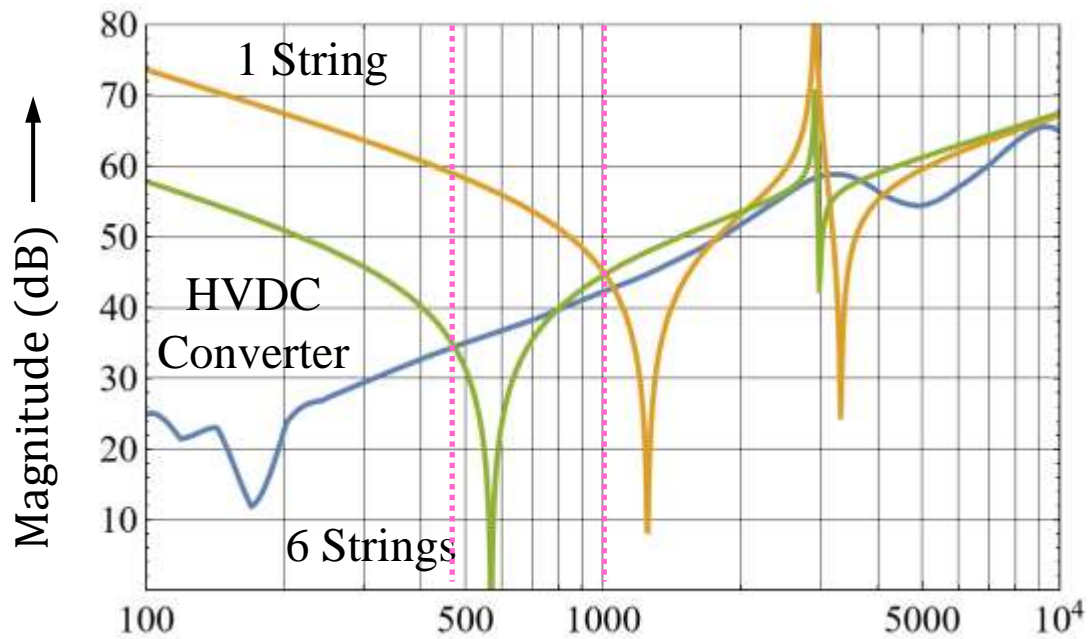
- Control Delay Turns Positive Damping by Current Control into Negative at High Frequency
- Resonance with Capacitive Impedance of the Network (above First Parallel Resonance)

$$Z_{ph}(s) = \frac{1}{2} \left[ sL + e^{-sT_d} \left( K_{p0} + \frac{K_{i0}}{s - j\omega_1} - jK_{d0} \right) \right]$$



# Energization of Offshore Cable Network

- Unterminated Offshore Cable Network Impedance is Capacitive
  - Resonance with HVDC Converter Impedance, Negative Damping Causes Instability
- Changing Startup Sequence May Solve the Problem or Just Shift the Frequency



# Grid-Forming Converter Impedance

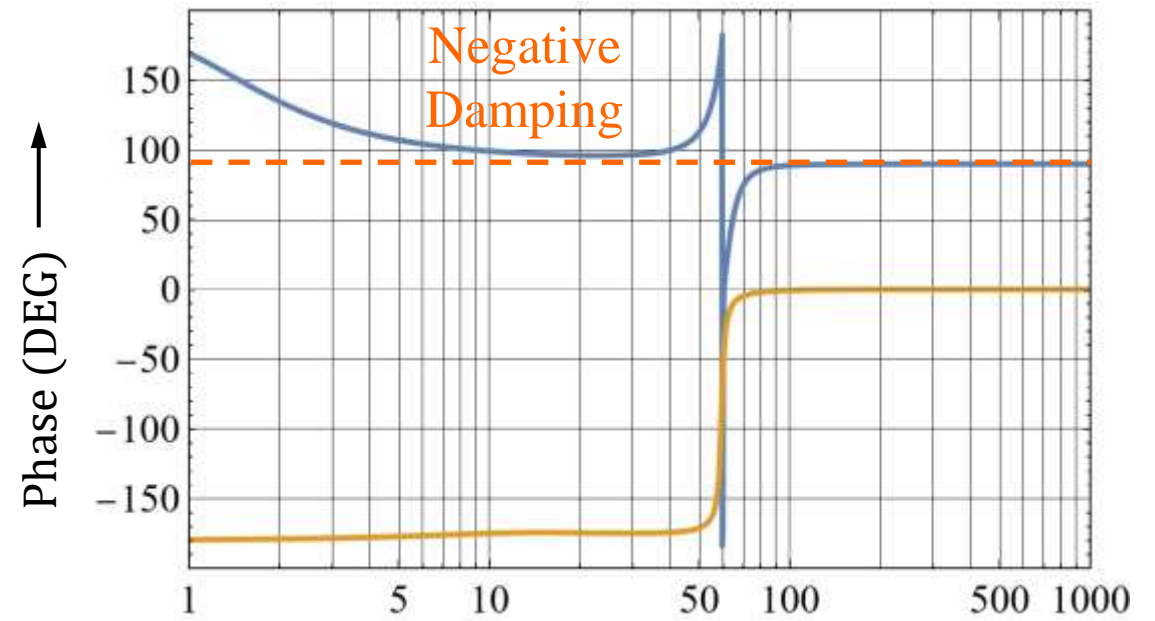
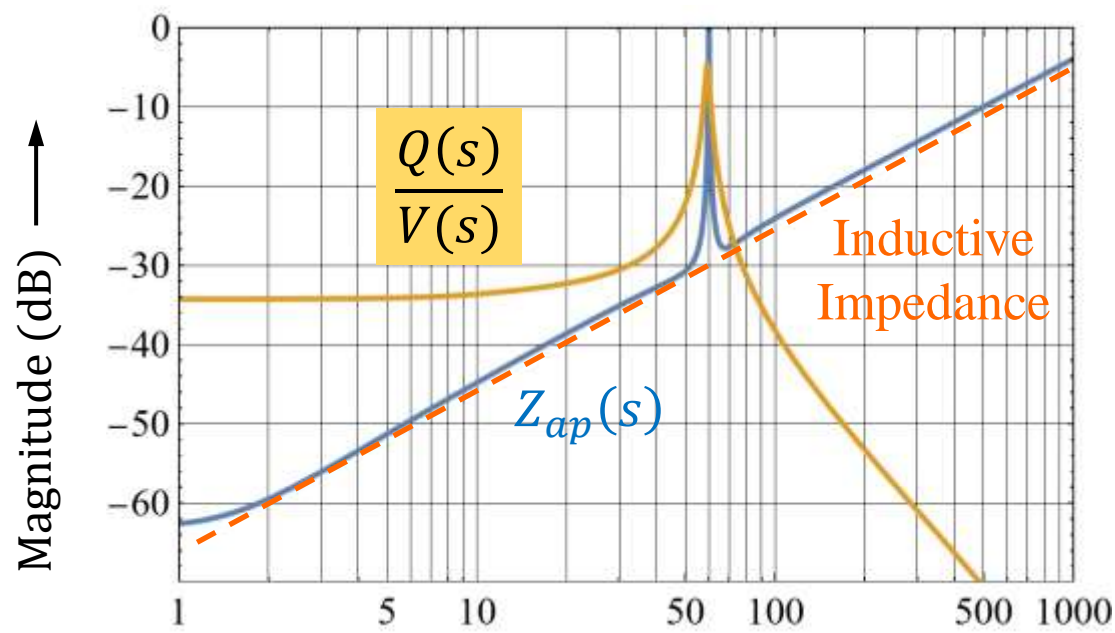
$$Z_{ap}(j\omega)$$

$$Z_{cp}(j\omega)$$

- Proportional Gain (P and Q Droop)
  - *P* Loop: Negative Damping Below  $\omega_1$ ; Positive Damping Above  $\omega_1$
  - *Q* Loop: Inductive Reactance
- Integral Gain (VSG Control)
  - *P* Loop: Capacitive Reactance
  - *Q* Loop: Negative Damping Below  $\omega_1$ ; Positive Damping Above  $\omega_1$

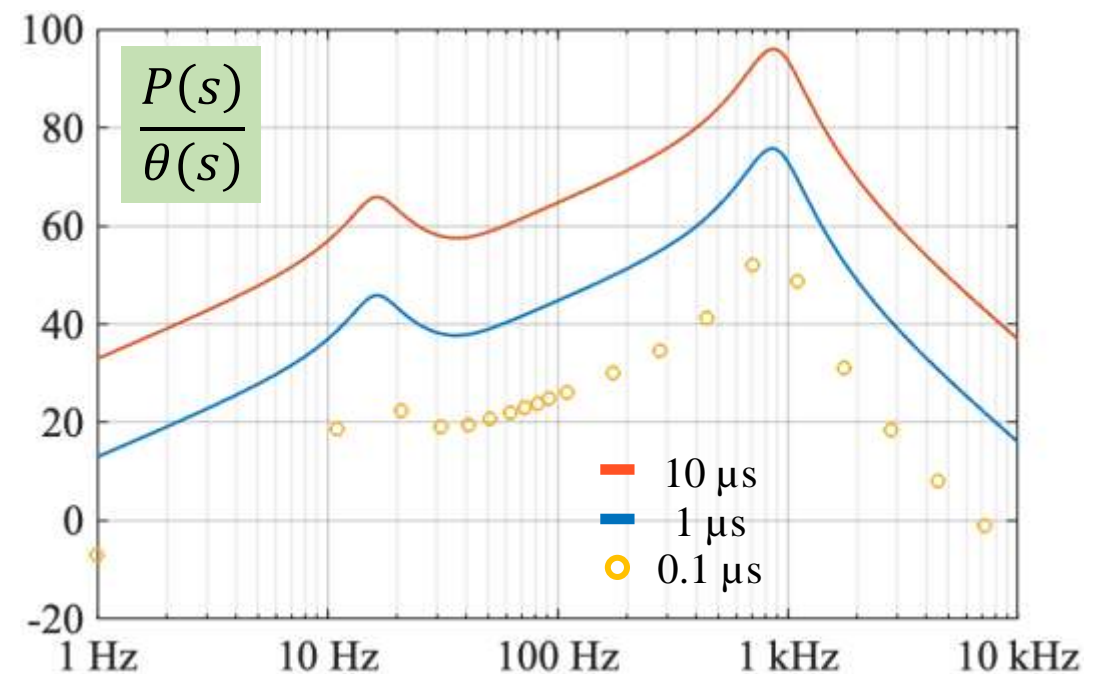
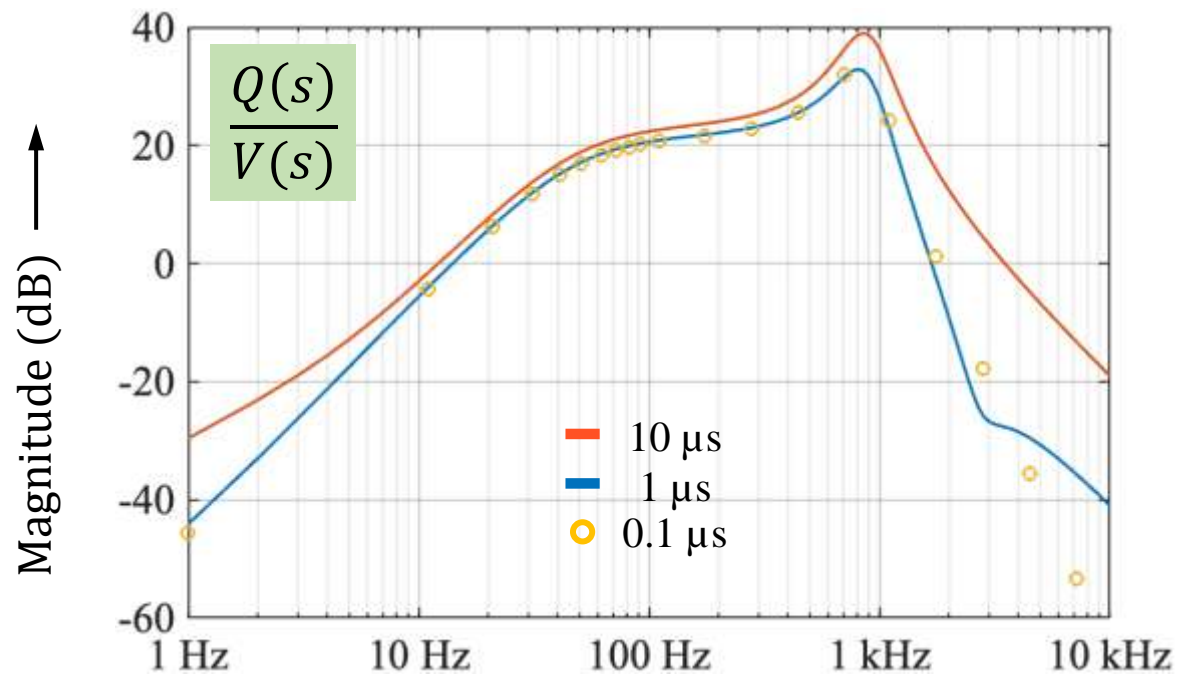
# Q-V and P-θ Response vs. Impedance

- Proposal to Characterize GFM by Scan of Q-V and P-θ Responses
- These Responses Provide no More Information than Impedances do
- Examination of Impedance is More Direct and Insightful



# Scan of Q-V and P-θ Responses

- Accurate Scan Requires 1 μs or Smaller Simulation Time Step
  - Converter Represented by Averaged Model
  - Problems May be Specific to Simulation Tool



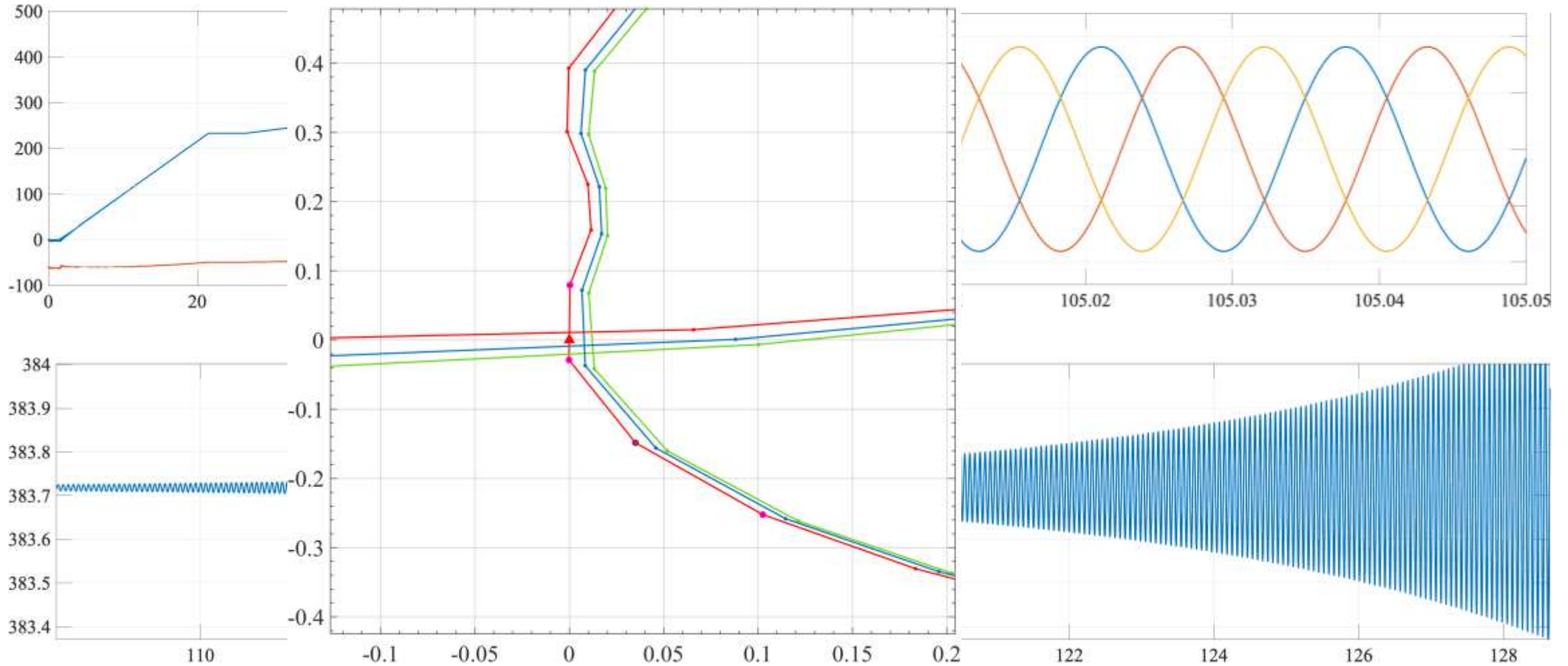


# Other Small-Signal Methods

- Methods Based on Specialized Models are Difficult to Scale
  - Dynamic Phasor Models; Harmonic State-Space Models
- State-Space Models are Difficult to Develop
  - Frequency-Dependent Transmission Line Parameters, Control Delay
  - Linearization of Converter Models
- DQ-Frame Methods have Limitations at Component & System Level
  - What Works for Electromechanical Models May not Work for EMT Models
- Be Careful with “Generalized/Unified/Transformed” Models

$$\begin{bmatrix} v_\alpha + jv_\beta \\ v_\alpha - jv_\beta \end{bmatrix} = \begin{bmatrix} Z_{\alpha\beta+} & \\ & Z_{\alpha\beta-} \end{bmatrix} \begin{bmatrix} i_\alpha + ji_\beta \\ i_\alpha - ji_\beta \end{bmatrix} \quad \begin{matrix} Z_{\alpha\beta+} = f(\mathbf{Z}_{dq}) \\ Z_{\alpha\beta-} = g(\mathbf{Z}_{dq}) \end{matrix} \quad \begin{matrix} v_\alpha \pm jv_\beta = v_{p,n} \\ i_\alpha \pm ji_\beta = i_{p,n} \end{matrix} \quad \begin{matrix} Z_{\alpha\beta+} = Z_p \\ Z_{\alpha\beta-} = Z_n \end{matrix}$$

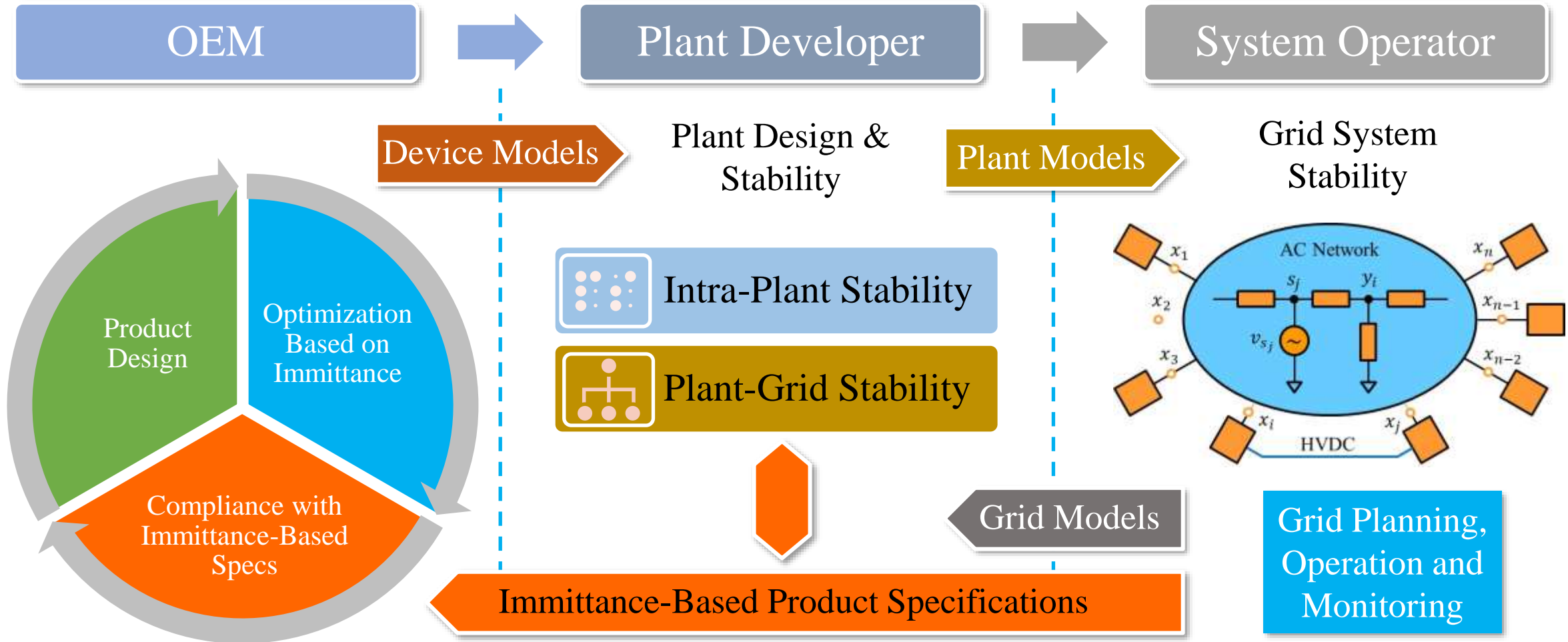
# Limitations of EMT Simulation



# Software Tool Development

- Run EMT Simulation to Perform Frequency Scan is Easy
- Plant-Grid Modeling & Analysis in SISO Form Requires no Tool
- Tools are Needed for Large System Analysis
  - Management of Component Models
  - Operation Conditions and Power Flow Calculation
  - System Modeling and Stability Analysis in MIMO Form
- EMT Tools with Fully Integrated Frequency and Time Domain Modeling, Analysis and Design Functions

# Summary



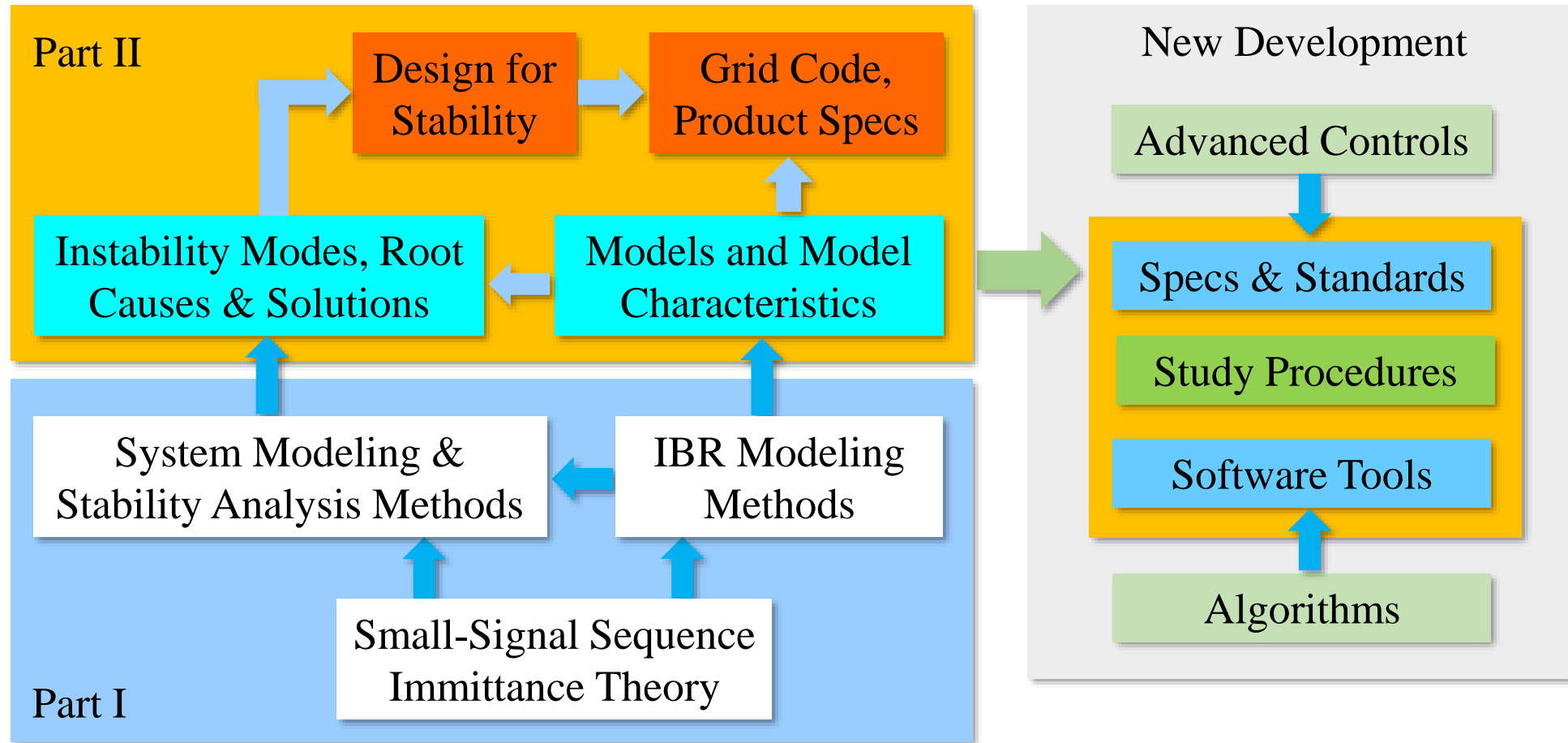
# Take This Course to Learn More

- Comprehensive & In-depth Coverage
- First Taught in 2023-2024
- 2<sup>nd</sup> Series to Start on Jan. 7, 2025
- 18 Weekly Lectures, Each 3 Hours
- All Lectures are Online
- **Early-Bird Registration Ends on Nov. 15, 2024**
- Questions? Email [jsun@rpi.edu](mailto:jsun@rpi.edu)
  - Also Available to Discuss Specific Technical Issues

[immittance-methods.net](http://immittance-methods.net)



# Course Organization and Approach



# Part I Lectures

1. Circuits and Control of Converters for Power System Application
2. Frequency-Domain Modeling and Analysis of Converters
3. Modeling of Voltage-Source Converters with Constant DC Bus Voltage
4. Modeling of Voltage-Source Converters with DC Bus Dynamics
5. **Impedance Modeling by Frequency Scan**
6. Modeling of Loads, Generators and Grid-Forming Converters
7. Stability at Converter-Grid Interface
8. Power System Stability with Grid-Following & Grid-Forming Converters
9. Stability of DC and Hybrid AC-DC Grids; Other Applications

# Part II Lectures

1. Immittance Responses and Effects of Control
2. PV Inverter and Wind Turbine Immittances
3. MMC-Based HVDC Converter Immittances
4. High-Frequency Instability – Modes and Root Causes
5. High-Frequency Instability – Solutions
6. Low-Frequency Instability – Modes and Root Causes
7. Low-Frequency Instability – Solutions
8. Multi-Converter System Instability – Modes and Analysis Methods
9. Design for System Stability; Future Development